CHAPTER 3

PAIR OF LINEAR EQUATION IN TWO VARIABLES

ONE MARK QUESTIONS

MULTIPLE CHOICE QUESTIONS

1. The value of k for which the system of linear equations $x+2y=3,\ 5x+ky+7=0$ is inconsistent is

(a) $-\frac{14}{3}$ (b) $\frac{2}{5}$

(c) 5 (d) 10

Ans : [Board 2020 OD Standard]

We have $x+2y-3=0$

and $5x+ky+7=0$

If system is inconsistent, then

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$$

From first two orders, we have

$$\frac{1}{5}=\frac{2}{k}\Rightarrow k=10$$

Thus (d) is correct option.

2. The value of k for which the system of equations $x+y-4=0$ and $2x+ky=3$, has no solution, is

(a) -2 (b) $\neq 2$

(c) 3 (d) 2

Ans : [Board 2020 Delhi Standard]

We have $x+y-4=0$

and $2x+ky-3=0$

Here, $\frac{a_1}{a_1}=\frac{1}{2},\ \frac{b_1}{b_2}=\frac{1}{k}$ and $\frac{c_1}{c_2}=\frac{-4}{-3}=\frac{4}{3}$

Since system has no solution, we have

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$$

$$\frac{1}{2}=\frac{1}{k}\neq\frac{4}{3}$$

$$k=2 \text{ and } k\neq\frac{3}{4}$$

Thus (d) is correct option.

3. For which value(s) of p, will the lines represented by the following pair of linear equations be parallel

$$3x-y-5=0$$

$$6x-2y-p=0$$

(a) all real values except 10 (b) 10

(c) 5/2 (d) 1/2

Ans :

We have, $3x-y-5=0$

and $6x-2y-p=0$

Here, $a_1=3,\ b_1=-1,\ c_1=-5$

and $a_2=6,\ b_2=-2,\ c_2=-p$

Since given lines are parallel,

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$$

$$\frac{-1}{-2}\neq\frac{-5}{-p}$$

$$p\neq 5\times 2\Rightarrow p\neq 10$$

4. The 2 digit number which becomes $\frac{5}{6}$th of itself when its digits are reversed. The difference in the digits of the number being 1, then the two digits number is

(a) 45 (b) 54

(c) 36 (d) None of these

Ans :

If the two digits are x and y, then the number is $10x+y$.

Now $\frac{5}{6}(10x+y)=10y+x$

Solving, we get $44x+55y$

$$\frac{x}{y}=\frac{5}{4}$$

Also $x-y=1$. Solving them, we get $x=5$ and $y=4$. Therefore, number is 54.

Thus (b) is correct option.

5. In a number of two digits, unit's digit is twice the tens digit. If 36 be added to the number, the digits are reversed. The number is

(a) 36 (b) 63

(c) 48 (d) 84

Ans :

Let x be units digit and y be tens digit, then number will be $10y+x$

Then, $x = 2y$ ...(1)

If 36 be added to the number, the digits are reversed, i.e number will be $10x+y$.

$$10y+x+36 = 10x+y$$

$$9x-9y = 36$$

$$x-y = 4 \quad ...(2)$$

Solving (1) and (2) we have $x=8$ and $y=4$.

Thus number is 48.

Thus (c) is correct option.

6. If $3x+4y:x+2y=9:4$, then $3x+5y:3x-y$ is equal to

(a) 4 : 1 (b) 1 : 4

(c) 7 : 1 (d) 1 : 7

Ans :

$$\frac{3x+4y}{x+2y} = \frac{9}{4}$$

Hence, $12x+16y = 9x+18y$

or $3x = 2y$

$$x = \frac{2}{3}y$$

Substituting $x=\frac{2}{3}y$ in the required expression we have

$$\frac{3x\frac{2}{3}y+5y}{3x\frac{2}{3}y-y} = \frac{7y}{y} = \frac{7}{1} = 7:1$$

Thus (c) is correct option.

7. A fraction becomes 4 when 1 is added to both the numerator and denominator and it becomes 7 when 1 is subtracted from both the numerator and denominator. The numerator of the given frac

(a) 2 (b) 3

(c) 5 (d) 15

Ans :

Let the fraction be $\frac{x}{y}$,

$$\frac{x+1}{y+1} = 4 \Rightarrow x = 4y+3 \quad ...(1)$$

and $$\frac{x-1}{y-1} = 7 \Rightarrow x = 7y-6 \quad ...(2)$$

Solving (1) and (2), we have $x = 15$, $y = 3$,

Thus (d) is correct option.

8. x and y are 2 different digits. If the sum of the two digit numbers formed by using both the digits is a perfect square, then value of $x+y$ is

(a) 10 (b) 11

(c) 12 (d) 13

Ans :

The numbers that can be formed are xy and yx. Hence, $(10x+y)+(10y+x) = 11(x+y)$. If this is a perfect square than $x+y = 11$.

9. The pair of equations $3^{x+y} = 81$, $81^{x-y} = 3$ has

(a) no solution

(b) unique solution

(c) infinitely many solutions

(d) $x = 2\frac{1}{8}, y = 1\frac{7}{8}$

Ans :

Given, $3^{x+y} = 81$

$$3^{x+y} = 3^4$$

$$x+y = 4 \quad ...(1)$$

and $81^{x-y} = 3$

$$3^{4(x-y)} = 3^1$$

$$4(x-y) = 1$$

$$x-y = \frac{1}{4} \quad ...(2)$$

Adding equation (1) and (2), we get

$$2x = 4+\frac{1}{4} = \frac{17}{4}$$

$$x = \frac{17}{8} = 2\frac{1}{8}$$

From equation (1), we get

$$y = \frac{15}{8} = 1\frac{7}{8}$$

Thus (d) is correct option.

10. The pair of linear equations $2kx+5y=7$, $6x-5y=11$

has a unique solution, if

(a) $k \neq -3$ (b) $k \neq \frac{2}{3}$

(c) $k \neq 5$ (d) $k \neq \frac{2}{9}$

Ans :

Given the pair of linear equations are

$$2kx + 5y - 7 = 0$$

and $$6x - 5y - 11 = 0$$

On comparing with

$$a_1x + b_1y + c_1 = 0$$

and $$a_2x + b_2y + c_2 = 0$$

we get, $$a_1 = 2k,\ b_1 = 5,\ c_1 = -7$$

and $$a_2 = 6,\ b_2 = -5,\ c_2 = -11$$

For unique solution,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$\frac{2k}{6} \neq \frac{5}{-5}$$

$$\frac{k}{3} = \neq -1$$

$$k \neq -3$$

Thus (a) is correct option.

11. The pair of equations $x + 2y + 5 = 0$ and $-3x - 6y + 1 = 0$ has

(a) a unique solution

(b) exactly two solutions

(c) infinitely many solutions

(d) no solution

Ans :

Given,equations are

$$x + 2y + 5 = 0$$

and $$-3x - 6y + 1 = 0$$

Here, $$a_1 = 1, b_1 = 2,\ c_1 = 5$$

and $$a_2 = -3,\ b_2 = -6,\ c_2 = 1$$

Now $$\frac{a_1}{a_2} = -\frac{1}{3},\ \frac{b_1}{b_2} = -\frac{2}{6} = -\frac{1}{3},\ \frac{c_1}{c_2} = \frac{5}{1}$$

Now, we observe that

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Hence, the pair of equations has no solution.

Thus (d) is correct option.

12. If a pair of linear equations is consistent, then the lines will be

(a) parallel

(b) always coincident

(c) intersecting or coincident

(d) always intersecting

Ans :

Condition for a consistent pair of linear equations

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

[intersecting lines having unique solution]

and $$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$ [coincident or dependent]

Thus (c) is correct option.

13. The pair of equations $y = 0$ and $y = -7$ has

(a) one solution

(b) two solutions

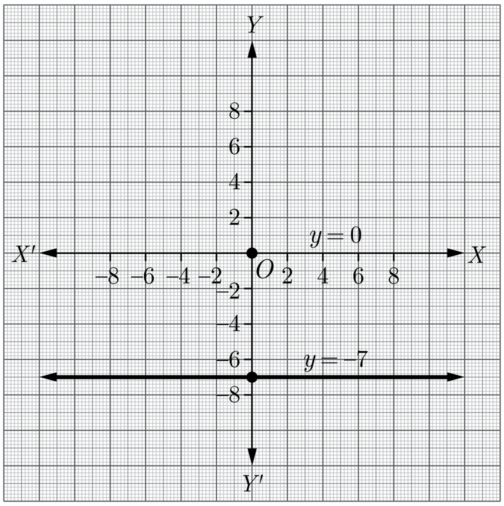
(c) infinitely many solutions

(d) no solution

Ans :

The given pair of equations are

$$y = 0 \quad y = -7$$



The pair of both equations are parallel to x-axis and we know that parallel lines never intersects. So, there is no solution of these lines.

Thus (d) is correct option.

14. The pair of equations $x = a$ and $y = b$ graphically represents lines which are

(a) parallel (b) intersecting at (b, a)

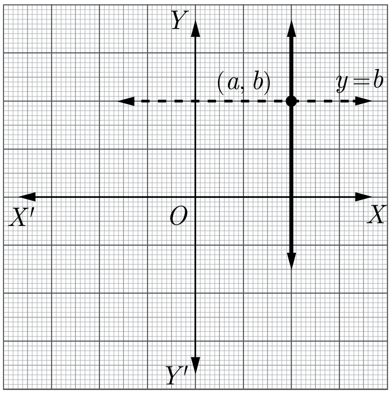
(c) coincident (d) intersecting at (a, b)

Ans :

The pair of equations

$$x = a$$

and $$y = b$$



Graphically represents lines which are intersecting at (a, b).
Thus (d) is correct option.

15. For what value of k, do the equations $3x - y + 8 = 0$ and $6x - ky = -16$ represent coincident lines ?

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$

(c) 2 (d) -2

Ans :

Given, equations,

$$3x - y + 8 = 0$$

and $$6x - ky + 16 = 0$$

Condition for coincident lines is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad ...(1)$$

Here, $a_1 = 3, b_1 = -1, c_1 = 8$

and $a_2 = 6, b_2 = -k, c_2 = 16$

From equation (1),

$$\frac{3}{6} = \frac{-1}{-k} = \frac{8}{16}$$

$$\frac{1}{k} = \frac{1}{2} \quad \left[\text{since } \frac{3}{6} = \frac{8}{16} = \frac{1}{2}\right]$$

$$k = 2$$

Thus (c) is correct option.

16. If the lines given by $3x + 2ky = 2$ and $2x + 5y + 1 = 0$ are parallel, then the value of k is

(a) $-\frac{5}{4}$ (b) $\frac{2}{5}$

(c) $\frac{15}{4}$ (d) $\frac{3}{2}$

Ans :

We have $3x + 2ky - 2 = 0$

and $2x + 5y + 1 = 0$

Condition for parallel lines is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \quad ...(i)$$

Here, $a_1 = 3, b_1 = 2k, c_1 = -2$

and $a_2 = 2, b_2 = 5, c_2 = 1$

From equation (i), we have

$$\frac{3}{2} = \frac{2k}{5} \neq \frac{-2}{1}$$

Considering, $\frac{3}{2} = \frac{2k}{5}$ $\left[\frac{3}{2} \neq \frac{-2}{1} \text{ in any case}\right]$

$$k = \frac{15}{4}$$

Thus (c) is correct option.

17. The value of c for which the pair of equations $cx - y = 2$ and $6x - 2y = 3$ will have is

(a) 3 (b) -3

(c) -12 (d) no value

Ans :

The given lines are, $cx - y = 2$

and $6x - 2y = 3$

Condition for infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad ...(i)$$

Here, $a_1 = c,\ b_1 = -1,\ c_1 = -2$

and $a_2 = 6,\ b_2 = -2,\ c_2 = -3$

From equation (i), $\frac{c}{6} = \frac{-1}{-2} = \frac{-2}{-3}$

Here, $\frac{c}{6} = \frac{1}{2}$

and $\frac{c}{6} = \frac{2}{3}$

$$c = 3$$

and $$c = 4$$

Since, c has different values.

Hence, for no value of c the pair of equations will have infinitely many solutions.

Thus (d) is correct option.

18. One equation of a pair of dependent linear equations $-5x+7y=2$ The second equation can be

(a) $10x+14y+4=0$ (b) $-10x-14y+4=0$

(c) $-10x+14y+4=0$ (d) $10x-14y=-4$

Ans :

For dependent linear equation,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Checking for option (a):

$$\frac{-5}{10} \neq \frac{7}{14}$$

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$ So, option (a) is rejected.

Checking for option (b):

$$\frac{-5}{-10} \neq \frac{7}{-14}$$

So, option (b) is also rejected.

Checking for option (c):

$$\frac{-5}{-10} = \frac{7}{14} \neq \frac{-2}{4}$$

So, option (b) is also rejected

Checking for option (d):

$$\frac{-5}{10} = \frac{7}{-14} = \frac{-2}{4}$$

Thus (d) is correct option.

19. If $x=a$ and $y=b$ is the solution of the equations $x-y=2$ and $x+y=4$, then the values of a and b are, respectively

(a) 3 and 5 (b) 5 and 3

(c) 3 and 1 (d) -1 and -3

Ans :

Since, $x=a$ and $y=b$ is the solution of the equations $x-y=2$ and $x+y=4$, then these values will satisfy that equation

$$a-b = 2 \quad ...(1)$$

and $$a+b = 4$$

Adding equations (1) and (2), we get

$$2a = 6$$

$$a = 3$$

Substituting $a = 3$ in equation (2),we have

$$3+b = 4 \Rightarrow b = 1$$

Thus $a = 3$ and $b = 1$.

Thus (c) is correct option.

20. Aruna has only ₹ 1 and ₹ 2 coins with her. If the total number of coins that she has is 50 and the amount of money with her is ₹ 75, then the number of ₹ 1 and ₹ 2 coins are, respectively

(a) 35 and 15 (b) 35 and 20

(c) 15 and 35 (d) 25 and 25

Ans :

Let number of ₹ 1 coins $= x$

and number of ₹ 2 coins $= y$

Now, by given conditions,

$$x+y = 50 \quad ...(1)$$

Also, $$x \times 1 + y \times 2 = 75$$

$$x+2y = 75 \quad ...(2)$$

Subtracting equation (1) form equation (2), we get

$$(x+2y)-(x+y) = 75-50$$

$$y = 25$$

From equation (i), $x = 75-2x(25)$

Then, $$x = 25$$

Thus (d) is correct option.

21. The father's age is six times his son's age. Four years hence, the age of the father will be four times his son's age. The present ages (in year) of the son and the father are, respectively.

(a) 4 and 24 (b) 5 and 30

(c) 6 and 36 (d) 3 and 24

Ans :

Let the present age of father $= x$ years

and the present age of son $= y$ years

Four years hence, it has relation by given condition

$$(x+4) = 4(y+4)$$

$$x-4y = 12 \quad ...(1)$$

As the father's age is six times his son's age, so we have

$$x = 6y \quad ...(2)$$

Putting the value of x from equation (2) in equation (1), we get

$$6x-4y = 12$$

$$2y = 12$$

$$y = 6$$

From equation (1), $x = 6\times 6$

Then, $x = 36$

Hence, present age of father is 36 year and age of son is 6 year.

Thus (c) is correct option.

22. **Assertion :** Pair of linear equations : $9x+3y+12=0$, $8x+6y+24=0$ have infinitely many solutions.

Reason : Pair of linear equations $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ have infinitely many solutions, if $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

From the given equations, we have

$$\frac{9}{18} = \frac{3}{6}=\frac{12}{24}$$

$$\frac{1}{2} = \frac{1}{2}=\frac{1}{2} \text{ i.e., } \frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Thus (a) is correct option.

23. **Assertion :** $x+y-4=0$ and $2x+ky-3=0$ has no solution if $k=2$.

Reason : $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ are consistent if $\frac{a_1}{a_2}\neq\frac{k_1}{k_2}$.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

For assertion, given equation has no solution if

$$\frac{1}{2} = \frac{1}{k}\neq\frac{-4}{-3} \text{ i.e. } \frac{4}{3}$$

$$k = 2\Big[\frac{1}{2}\neq\frac{4}{3}\text{holds}$$

Assertion is true.

Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

Thus (b) is correct option.

Fill in the Blank Questions

24. If the lines intersect at a point, then that point gives the unique solution of the two equations. In this case, the pair of equations is ....

Ans :

consistent

25. An equation whose degree is one is known as a .... equation.

Ans :

linear

26. If the lines are parallel, then the pair of equations has no solution. In this case, the pair of equations i

Ans :

inconsistent

27. A pair of linear equations has .... solution(s) if it is represented by intersecting lines graphically.

Ans :

unique

28. Every solution of a linear equation in two variables is a point on the .... representing it.

Ans :

line

29. If a pair of linear equations has infinitely many solutions, then its graph is represented by a pair of .... lines.

Ans :

coincident

30. A pair of linear equations is .... if it has no solution.

Ans :

inconsistent

31. A pair of .... lines represent the pair of linear equations having no solution.

Ans :

parallel

32. If a pair of linear equations has solution, either a unique or infinitely many, then it is said to be ....

Ans :

consistent

33. If the equations $kx-2y=3$ and $3x+y=5$ represent two intersecting lines at unique point, then the value of k is ....

Ans :

For unique solution

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Here, $a_1=k$, $b_1=-2$, $a_2=3$ and $b_2=1$

Now $\quad \frac{k}{3} \neq -\frac{2}{1}$

or, $\quad k \neq -6$

VERY SHORT ANSWER QUESTIONS

34. Find whether the pair of linear equations $y=0$ and $y=-5$ has no solution, unique solution or infinitely many solutions.

Ans :

The given variable y has different values. Therefore the pair of equations $y=0$ and $y=-5$ has no solution.

35. If $am=bl$, then find whether the pair of linear equations $ax+by=c$ and $lx+my=n$ has no solution, unique solution or infinitely many solutions.

Ans :

Since, $am=bl$, we have

$$\frac{a}{1}=\frac{b}{m} \neq \frac{c}{n}$$

Thus, $ax+by=c$ and $lx+my=n$ has no solution.

36. If $ad \neq bc$, then find whether the pair of linear equations $ax+by=p$ and $cx+dy=q$ has no solution, unique solution or infinitely many solutions.

Ans :

Since $\quad ad \neq bc$ or $\frac{a}{c} \neq \frac{b}{d}$

Hence, the pair of given linear equations has unique solution.

37. Two lines are given to be parallel. The equation of one of the lines is $4x+3y=14$, then find the equation of the second line.

Ans :

The equation of one line is $4x+3y=14$. We know that if two lines $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ are parallel, then

$$\frac{a_1}{a_2}=\frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

or $\quad \frac{4}{a_2}=\frac{3}{b_2} \neq \frac{c_1}{c_2} \Rightarrow \frac{a_2}{b_2}=\frac{4}{3}=\frac{12}{9}$

Hence, one of the possible, second parallel line is $12x+9y=5$.

TWO MARKS QUESTIONS

38. Find the value(s) of k so that the pair of equations $x+2y=5$ and $3x+ky+15=0$ has a unique solution.

Ans : [Board 2019 OD]

We have $\quad x+2y-5=0 \quad$ ...(1)

and $\quad 3x+ky+15=0 \quad$ ...(2)

Comparing equation (1) with $a_1x+b_1y+c_1=0$, and equation (2) with $a_2x+b_2y+c_2=0$, we get

$$a_1=1,\ a_2=3,\ b_1=2,\ b_2=k,\ c_1=-5 \text{ and } c_2=15$$

Since, given equations have unique solution, So,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

i.e. $\quad \frac{1}{3} \neq \frac{2}{k}$

$$k \neq 6$$

Hence, for all values of k except 6, the given pair of equations have unique solution.

39. If $2x + y = 23$ and $4x - y = 19$, find the value of $(5y - 2x)$ and $(\frac{y}{x} - 2)$.

Ans : [Board 2020 OD Standard]

We have $2x + y = 23$ ...(1)

$4x - y = 19$ ...(2)

Adding equation (1) and (2), we have

$$6x = 42 \Rightarrow x = 7$$

Substituting the value of x in equation (1), we get

$$14 + y = 23$$

$$y = 23 - 14 = 9$$

Hence, $5y - 2x = 5 \times 9 - 2 \times 7$

$$= 45 - 14 = 31$$

and $\frac{y}{x} - 2 = \frac{9}{7} - 2 = \frac{9 - 14}{7} = \frac{-5}{7}$

40. Find whether the lines represented by $2x + y = 3$ and $4x + 2y = 6$ are parallel, coincident or intersecting.

Ans : [Board Term-1 2016, MV98HN3]

Ans :

Here $a_1 = 2, b_1 = 1, c_1 = -3$ and $a_1 = 4, b_2 = 2, c_2 = -6$

If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

then the lines are parallel.

Clearly $\frac{2}{4} = \frac{1}{2} = \frac{3}{6}$

Hence lines are coincident.

41. Find whether the following pair of linear equation is consistent or inconsistent:

$3x + 2y = 8$, $6x - 4y = 9$

Ans : [Board Term-1 2016]

We have $\frac{3}{6} \neq \frac{2}{-4}$

i.e., $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence, the pair of linear equation is consistent.

42. Is the system of linear equations $2x + 3y - 9 = 0$ and $4x + 6y - 18 = 0$ consistent? Justify your answer.

Ans : [Board Term-1 2012]

For the equation $2x + 3y - 9 = 0$ we have

$$a_2 = 2, b_1 = 3 \text{ and } c_1 = -9$$

and for the equation, $4x + 6y - 18 = 0$ we have

$$a_2 = 4, b_2 = 6 \text{ and } c_2 = -18$$

Here $\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$

$$\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$$

and $\frac{c_1}{c_2} = \frac{-9}{-18} = \frac{1}{2}$

Thus $\frac{c_1}{c_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Hence, system is consistent and dependent.

43. Given the linear equation $3x + 4y = 9$. Write another linear equation in these two variables such that the geometrical representation of the pair so formed is:

(1) intersecting lines

(2) coincident lines.

Ans : [Board Term-1 2016, Set-O4YP6G7]

(1) For intersecting lines $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

So, one of the possible equation $3x - 5y = 10$

(2) For coincident lines $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

So, one of the possible equation $6x + 8y = 18$

44. For what value of p does the pair of linear equations given below has unique solution ?

$4x + py + 8 = 0$ and $2x + 2y + 2 = 0$.

Ans : [Board Term-1 2012]

We have $4x + py + 8 = 0$

$$2x + 2y + 2 = 0$$

The condition of unique solution, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence, $\frac{4}{2} \neq \frac{p}{2}$ or $\frac{2}{1} \neq \frac{p}{2}$

Thus $p \neq 4$. The value of p is other than 4 it may be 1,2,3, $-$ 4.....etc.

45. For what value of k, the pair of linear equations $kx - 4y = 3$, $6x - 12y = 9$ has an infinite number of solutions ?

Ans : [Board Term-1 2012]

We have $kx - 4y - 3 = 0$

and $6x - 12y - 9 = 0$

where, $a_1 = k, b_1 = 4, c_1 = -3$

$a_2 = 6, b_2 = -12, c_2 - 9$

Condition for infinite solutions:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{k}{6} = \frac{-4}{-12} = \frac{3}{9}$$

Hence, $k = 2$

46. For what value of k, $2x + 3y = 4$ and $(k+2)x + 6y = 3k + 2$ will have infinitely many solutions ?

Ans : [Board Term-1 2012]

We have $2x + 3y - 4 = 0$

and $(k+2)x + 6y - (3k+2) = 0$

Here $a_1 = 2, b_1 = 3, c_1 = -4$

and $a_2 = k+2, b_2 = 6, c_3 = -(3k+2)$

For infinitely many solutions

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

or, $$\frac{2}{k+2} = \frac{3}{6} = \frac{4}{3k+2}$$

From $\frac{2}{k+2} = \frac{3}{6}$ we have

$3(k+2) = 2 \times 6 \Rightarrow (k+2) = 4 \Rightarrow k = 2$

From $\frac{3}{6} = \frac{4}{3k+2}$ we have

$3(3k+2) = 4 \times 6 \Rightarrow (3k+2) = 8 \Rightarrow k = 2$

Thus $k = 2$

47. For what value of k, the system of equations $kx + 3y = 1,\ 12x + ky = 2$ has no solution.

Ans : [Board Term-1 2011, NCERT]

The given equations can be written as
$kx + 3y - 1 = 0$ and $12x + ky - 2 = 0$

Here, $a_1 = k, b_1 = 3, c_1 = -1$

and $a_2 = 12, b_2 = k, c_2 = -2$

The equation for no solution if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

or, $$\frac{k}{12} = \frac{3}{k} \neq \frac{-1}{-2}$$

From $\frac{k}{12} = \frac{3}{k}$ we have $k^2 = 36 \Rightarrow k \pm 6$

From $\frac{3}{k} \neq \frac{-1}{-2}$ we have $k \neq 6$

Thus $k = -6$

48. Solve the following pair of linear equations by cross multiplication method:

$$x + 2y = 2$$

$$x - 3y = 7$$

Ans : [Board Term-1 2016]

We have $x + 2y - 2 = 0$

$x - 3y - 7 = 0$

Using the formula

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

we have $$\frac{x}{-14-6} = \frac{y}{-2+7} = \frac{1}{-3-2}$$

$$\frac{x}{-20} = \frac{y}{5} = \frac{-1}{5}$$

$$\frac{x}{-20} = \frac{-1}{5} \Rightarrow x = 4$$

$$\frac{y}{5} = \frac{-1}{5} \Rightarrow y = -1$$

49. Solve the following pair of linear equations by substitution method:

$$3x + 2y - 7 = 0$$
$$4x + y - 6 = 0$$

Ans : [Board Term-1 2015]

We have $3x + 2y - 7 = 0$ ...(1)

$4x + y - 6 = 0$ ...(2)

From equation (2), $y = 6 - 4x$ ...(3)

Putting this value of y in equation (1) we have

$$3x + 2(6 - 4x) - 7 = 0$$
$$3x + 12 - 8x - 7 = 0$$
$$5 - 5x = 0$$
$$5x = 5$$

Thus $x = 1$

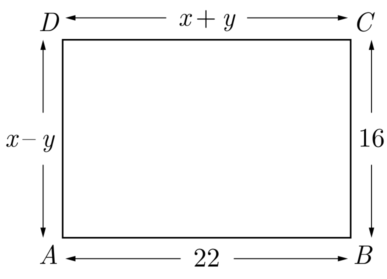
Substituting this value of x in (2), we obtain,

$$y = 6 - 4 \times 1 = 2$$

Hence, values of x and y are 1 and 2 respectively.

50. In the figure given below, $ABCD$ is a rectangle. Find the values of x and y.

Ans : [Board Term-1 2012, Set-30]



From given figure we have

$x + y = 22$ ...(1)

and $x - y = 16$

Adding (1) and (2), we have

$$2x = 38$$
$$x = 19$$

Substituting the value of x in equation (1), we get

$$19 + y = 22$$
$$y = 22 - 19 = 3$$

Hence, $x = 19$ and $y = 3$.

51. Solve : $99x + 101y = 499$, $101x + 99y = 501$

Ans : [Board Term-1 2012, Set-55]

We have $99x + 101y = 499$ ...(1)

$101x + 99y = 501$ ...(2)

Adding equation (1) and (2), we have

$$200x + 200y = 1000$$

$x + y = 5$ ...(3)

Subtracting equation (2) from equation (1), we get

$$-2x + 2y = -2$$

$x - y = 1$ ...(4)

Adding equations (3) and (4), we have

$$2x = 6 \Rightarrow x = 3$$

Substituting the value of x in equation (3), we get

$$3 + y = 5 \Rightarrow y = 2$$

52. Solve the following system of linear equations by substitution method:

$$2x - y = 2$$
$$x + 3y = 15$$

Ans : [Board Term-1 2012]

We have $2x - y = 2$ ...(1)

$x + 3y = 15$ ...(2)

From equation (1), we get $y = 2x - 2$ ...(3)

Substituting the value of y in equation (2),

$$x + 6x - 6 = 15$$

or, $7x = 21 \Rightarrow x = 3$

Substituting this value of x in (3), we get

From equation (1), we have

$$y = 2 \times 3 - 2 = 4$$

$$x = 3 \text{ and } y = 4$$

53. Find the value(s) of k for which the pair of Linear equations $kx + y = d^2$ and $x + ky = 1$ have infinitely many solutions.

Ans : [Board Term-1 2017]

We have $\quad kx + y = k^2$

and $\quad x + ky = 1$

$$\frac{a_1}{a_2} = \frac{k}{1}, \frac{b_1}{b_2} = \frac{1}{k}, \frac{c_1}{c_2} = \frac{k^2}{1}$$

For infinitely many solution

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{k}{1} = \frac{1}{k} = \frac{k^2}{1} = k^2 = 1$$

$$k = \pm 1$$

THREE MARKS QUESTIONS

54. Solve the following system of equations.

$$\frac{21}{x} + \frac{47}{y} = 110, \frac{47}{x} + \frac{21}{y} = 162, x, y \neq 0$$

Ans :

We have $$\frac{21}{x} + \frac{47}{y} = 110$$

$$\frac{47}{x} + \frac{21}{y} = 162$$

Let $\frac{1}{x} = u$ and $\frac{1}{y} = v$. then given equation become

$$21u + 47v = 110 \quad ...(1)$$

and $$47u + 21v = 162 \quad ...(2)$$

Adding equation (1) and (2) we get

$$68u + 68v = 272$$

$$u + v = 4 \quad ...(3)$$

Subtracting equation (1) from (2) we get

$$26u - 26v = 52$$

$$u - v = 2 \quad ...(4)$$

Adding equation (3) and (4), we get

$$2u = 6 \Rightarrow u = 3$$

Substituting $u = 3$ in equation (3), we get $v = 1$.

Thus $x = \frac{1}{u} = \frac{1}{3}$ and $y = \frac{1}{v} = \frac{1}{1} = 1$

55. A fraction becomes $\frac{1}{3}$ when 2 is subtracted from the numerator and it becomes $\frac{1}{2}$ when 1 is subtracted from the denominator- Find the fraction.

Ans : [Board 2019 Delhi]

Let the fraction be $\frac{x}{y}$. According to the first condition,

$$\frac{x-2}{y} = \frac{1}{3}$$

$$3x - 6 = y$$

$$y = 3x - 6 \quad ...(1)$$

According to the second condition,

$$\frac{x}{y-1} = \frac{1}{2}$$

$$2x = y - 1$$

$$y = 2x + 1 \quad ...(2)$$

From equation (1) and (2), we have

$$3x - 6 = 2x + 1 \Rightarrow x = 7$$

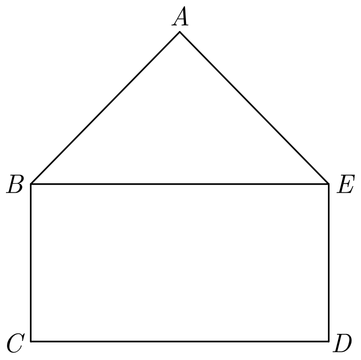
Substitute value of x in equation (1), we get

$$y = 3(7) - 6$$

$$= 21 - 6 = 15$$

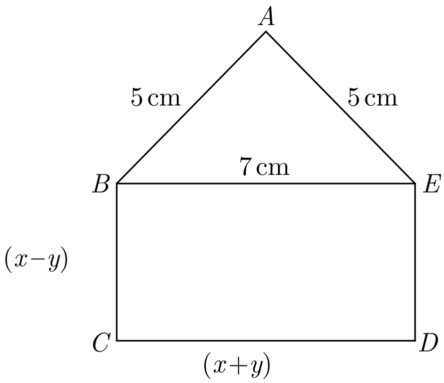
Hence, fraction is $\frac{7}{15}$.

56. In the figure, $ABCDE$ is a pentagon with $BE \parallel CD$ and $BC \parallel DE$. BC is perpendicular to CD. $AB = 5$ cm, $AE = 5$ cm, $BE = 7$ cm, $BC = x - y$ and $CD = x + y$. If the perimeter of $ABCDE$ is 27 cm. Find the value of x and y, given $x, y \neq 0$.



Ans : [Board 2020 SQP Standard]

We have redrawn the given figure as shown below.



We have $CD = BE$

$$x + y = 7 \quad ...(1)$$

Also, perimeter of $ABCDE$ is 27 cm, thus

$$AB + BC + CD + DE + AE = 27$$

$$5 + (x - y) + (x + y) + (x - y) + 5 = 27$$

$$3x - y = 17 \quad ...(2)$$

Adding equation (1) and (2) we have

$$4x = 24 \Rightarrow x = 6$$

Substituting $x = 6$ in equation (1) we obtain

$$y = 7 - x = 7 - 6 = 1$$

Thus $x = 6$ and $y = 1$.

57. Half the perimeter of a rectangular garden, whose length is 4 m more then its width, is 36 m. Find the dimensions of garden.

Ans : [Board Term-1 2013]

Let the length of the garden be x m and its width be y m.

Perimeter of rectangular garden

$$p = 2(x + y)$$

Since half perimeter is given as 36 m,

$$(x + y) = 36 \quad ...(1)$$

Also, $x = y + 4$

or $x - y = 4 \quad ...(2)$

For $x + y = 36$

$$y = 36 - x$$

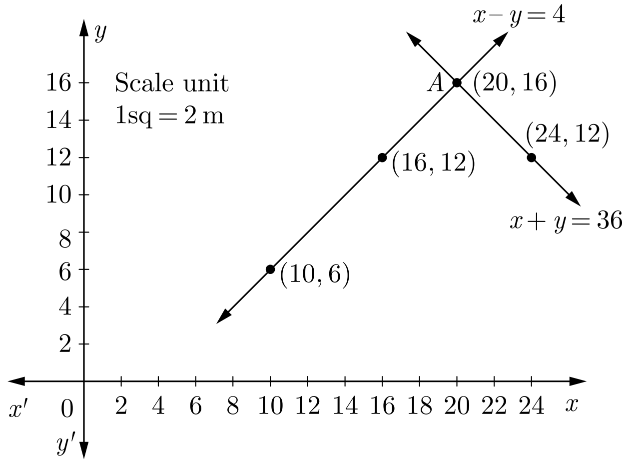
x	20	24
y	16	12

For $x - y = 4$

or, $y = x - 4$

x	10	16	20
y	6	12	16

Plotting the above points and drawing lines joining them, we get the following graph. we get two lines intersecting each other at (20, 16)



Hence, length is 20 m and width is 16 m.

58. Given the linear equation $2x + 3y - 8 = 0$, write another linear equation in two variables such that the geometrical representation of the pair so formed is :

(a) intersecting lines
(b) parallel lines
(c) coincident lines.

Ans : [Board Term-1 2014, Set-B]

Given, linear equation is $2x + 3y - 8 = 0 \quad ...(1)$

(a) For intersecting lines, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

To get its parallel line one of the possible equation may be taken as

$$5x + 2y - 9 = 0 \quad (2)$$

(b) For parallel lines, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

One of the possible line parallel to equation (1) may be taken as

$$6x + 9y + 7 = 0$$

(c) For coincident lines, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

To get its coincident line, one of the possible equation may be taken as

$$4x + 6y - 16 = 0$$

59. Solve the pair of equations graphically :

$4x - y = 4$ and $3x + 2y = 14$

Ans : [Board Term-1 2014]

We have $4x - y = 4$

or, $y = 4x - 4$

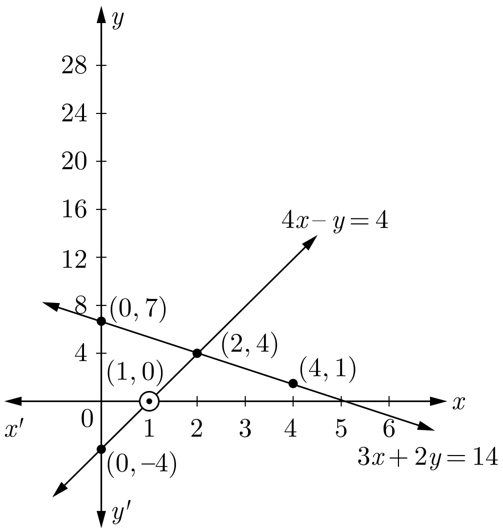
x	0	1	2
y	-4	0	4

and $3x + 2y = 14$

or, $y = \frac{14 - 3x}{2}$

x	0	2	4
y	7	4	1

Plotting the above points and drawing lines joining them, we get the following graph. We get two obtained lines intersect each other at (2, 4).



Hence, $x = 2$ and $y = 4$.

60. Determine the values of m and n so that the following system of linear equation have infinite number of solutions :

$$(2m-1)x + 3y - 5 = 0$$

$$3x + (n-1)y - 2 = 0$$

Ans : [Board Term-1 2013, VKH6FFC; 2011, Set-66

We have $(2m-1)x + 3y - 5 = 0$ ...(1)

Here $a_1 = 2m - 1, b_1 = 3, c_1 = -5$

$3x + (n-1)y - 2 = 0$ ...(2)

Here $a_2 = 3, b_2 = (n-1), c_2 = -2$

For a pair of linear equations to have infinite number of solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{2m-1}{3} = \frac{3}{n-1} = \frac{5}{2}$$

or $2(2m-1) = 15$ and $5(n-1) = 6$

Hence, $m = \frac{17}{4}, n = \frac{11}{5}$

61. Find the values of α and β for which the following pair of linear equations has infinite number of solutions : $2x + 3y = 7$; $2\alpha x + (\alpha + \beta)y = 28$.

Ans : [Board Term-1 2011]

We have $2x + 3y = 7$ and $2\alpha x + (\alpha + \beta)y = 28$.

For a pair of linear equations to be consistent and having infinite number of solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{2}{2\alpha} = \frac{3}{\alpha + \beta} = \frac{7}{28}$$

$$\frac{2}{2\alpha} = \frac{7}{28}$$

$$2\alpha \times 7 = 28 \times 2 \Rightarrow \alpha = 4$$

$$\frac{3}{\alpha + \beta} = \frac{7}{28}$$

$$7(\alpha + \beta) = 28 \times 3$$

$$\alpha + \beta = 12$$

$$\beta = 12 - \alpha = 12 - 4 = 8$$

Hence $\alpha = 4$, and $\beta = 8$

62. Represent the following pair of linear equations graphically and hence comment on the condition of consistency of this pair.

$x - 5y = 6$ and $2x - 10y = 12$.

Ans : [Board Term-1 2011]

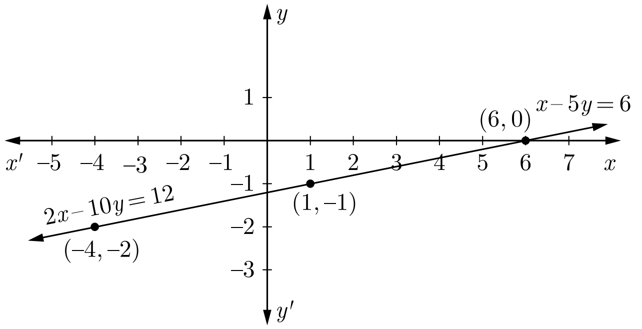
We have $x - 5y = 6$ or $x = 5y + 6$

x	6	1	-4
y	0	-1	-2

and $2x - 10y = 12$ or $x = 5y + 6$

x	6	1	-4
y	0	-1	-2

Plotting the above points and drawing lines joining them, we get the following graph.



Since the lines are coincident, so the system of linear equations is consistent with infinite many solutions.

63. For what value of p will the following system of equations have no solution ?

$(2p-1)x+(p-1)y = 2p+1;\ y+3x-1=0$

Ans : [Board Term-1 2011, Set-28]

We have $$(2p-1)x+(p-1)y-(2p+1) = 0$$

Here $a_1 = 2p-1, b_1 = p-1$ and $c_1 = -(2p+1)$

Also $$3x+y-1 = 0$$

Here $a_2 = 3, b_2 = 1$ and $c_2 = -1$

The condition for no solution is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\frac{2p-1}{3} = \frac{p-1}{1} \neq \frac{2p+1}{1}$$

From $\frac{2p-1}{3} = \frac{p-1}{1}$ we have

$$3p-3 = 2p-1$$

$$3p-2p = 3-1$$

$$p = 2$$

From $\frac{p-1}{1} \neq \frac{2p+1}{1}$ we have

$$p-1 \neq 2p+1 \text{ or } 2p-p \neq -1-1$$

$$p \neq -2$$

From $\frac{2p-1}{3} \neq \frac{2p+1}{1}$ we have

$$2p-1 \neq 6p+3$$

$$4p \neq -4$$

$$p \neq -1$$

Hence, system has no solution when $p = 2$

64. Find the value of k for which the following pair of equations has no solution :

$x+2y=3, (k-1)x+(k+1)y=(k+2)$.

Ans : [Board Term-1 2011, Set-52]

For $x+2y=3$ or $x+2y-3=0$,

$$a_1 = 1, b_1 = 2, c_1 = -3$$

For $(k-1)x+(k+1)y=(k+2)$

or $(k-1)x+(k+1)y-(k-2)=0$

$$a_2 = (k-1), b_2 = (k+1), c_2 = -(k+2)$$

For no solution, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$$\frac{1}{k-1} = \frac{2}{k+1} \neq \frac{3}{k+2}$$

From $\frac{1}{k-1} = \frac{2}{k+1}$ we have

$$k+1 = 2k-2$$

$$3 = k$$

Thus $k = 3$.

65. Sum of the ages of a father and the son is 40 years. If father's age is three times that of his son, then find their respective ages.

Ans : [Board Term-1 2015]

Let age of father and son be x and y respectively.

$$x+y = 40 \quad ...(1)$$

$$x = 3y \quad ...(2)$$

Solving equations (1) and (2), we get

$$x = 30 \text{ and } y = 10$$

Ages are 30 years and 10 years.

66. Solve using cross multiplication method:

$$5x+4y-4 = 0$$

$$x-12y-20 = 0$$

Ans : [Board Term-1 2015]

We have $$5x+4y-4 = 0 \quad ...(1)$$

$$x-12y-20 = 0 \quad ...(2)$$

By cross-multiplication method,

$$\frac{x}{b_2c_1-b_1c_2} = \frac{y}{c_1a_2-c_2a_1} = \frac{1}{b_1b_2-a_2b_1}$$

$$\frac{x}{-80-48} = \frac{y}{-4+100} = \frac{1}{-60-4}$$

$$\frac{x}{-128} = \frac{y}{96} = \frac{1}{64}$$

$$\frac{x}{-128} = \frac{1}{-64} \Rightarrow x = 2$$

and $$\frac{y}{96} = \frac{1}{-64} \Rightarrow y = \frac{-3}{2}$$

Hence, $x = 2$ and $y = \frac{-3}{2}$

67. The Present age of the father is twice the sum of the ages of his 2 children. After 20 years, his age will be equal to the sum of the ages of his children. Find the age of the father.

Ans : [Board Term-1 2012, Set-39]

Let the sum of the ages of the 2 children be x and the age of the father be y years.

Now $$y = 2x$$

$$2x - y = 0 \qquad ...(1)$$

and $$20 + y = x + 40$$

$$x - y = -20 \qquad ...(2)$$

Subtracting (2) from (1), we get

$$x = 20$$

From(1), $$y = 2x = 2 \times 20 = 40$$

Hence, the age of the father is 40 years.

68. A part of monthly hostel charge is fixed and the remaining depends on the number of days one has taken food in the mess. When Swati takes food for 20 days, she has to pay Rs. 3,000 as hostel charges whereas Mansi who takes food for 25 days Rs. 3,500 as hostel charges. Find the fixed charges and the cost of food per day.

Ans : [Board Term-1 2016, 2015]

Let fixed charge be x and per day food cost be y

$$x + 20y = 3000 \qquad ...(1)$$

$$x + 25y = 3500 \qquad ...(2)$$

Subtracting (1) from (2) we have

$$5y = 500 \Rightarrow y = 100$$

Substituting this value of y in (1), we get

$$x + 20(100) = 3000$$

$$x = 1000$$

Thus $x = 1000$ and $y = 100$

Fixed charge and cost of food per day are Rs. 1,000 and Rs. 100.

69. Solve for x and y :

$$\frac{x}{2} + \frac{2y}{3} = -1$$

$$x - \frac{y}{3} = 3$$

Ans :

We have $$\frac{x}{2} + \frac{2y}{3} = -1$$

$$3x + 4y = -6 \qquad ...(1)$$

and $$\frac{x}{1} - \frac{y}{3} = 3$$

$$3x - y = 9 \qquad ...(2)$$

Subtracting equation (2) from equation (1), we have

$$5y = -15 \Rightarrow y = -1$$

Substituting $y = -3$ in eq (1), we get

$$3x + 4(-3) = -6$$

$$3x - 12 = -6$$

$$3x = 12 - 6 \Rightarrow x = 2$$

Hence $x = 2$ and $y = -3$.

70. Solve the following pair of linear equations by the substitution and cross - multiplication method :

$$8x + 5y = 9$$

$$3x + 2y = 4$$

Ans : [Board Term-1 2015, SYFH4D]

We have $$8x + 5y = 9$$

or, $$8x + 5y - 9 = 0 \qquad ...(1)$$

and $$3x + 2y = 4$$

or, $$3x + 2y - 4 = 0 \qquad ...(2)$$

Comparing equation (1) and (2) with $ax + by + c = 0$,

$$a_1 = 8, b_1 = 5, c_1 = -9$$

and $a_2 = 3, b_2 = 2, c_2 = -4$

By cross-multiplication method,

$$\frac{x}{b_2 c_1 - b_1 c_2} = \frac{y}{c_1 a_2 - c_2 a_1} = \frac{1}{a_1 b_2 - b_2 b_1}$$

$$\frac{x}{\{(5)(-4)-(2)(-9)\}} = \frac{y}{\{(-9)(3)-(-4)(8)\}}$$

$$= \frac{1}{\{8 \times 2 - 3 \times 5}$$

or, $\frac{x}{-2} = \frac{1}{1}$ and $\frac{y}{5} = \frac{1}{1}$

$x = -2$ and $y = 5$

We use substitution method.
From equation (2), we have

$$3x = 4 - 2y$$

or, $$x = \frac{4-2y}{3} \quad ...(3)$$

Substituting this value of y in equation (3) in (1), we get

$$8\left(\frac{4-2y}{3}\right) + 5y = 9$$

$$32 - 16y + 15y = 27$$

$$-y = 27 - 32$$

Thus $y = 5$

Substituting this value of y in equation (3)

$$x = \frac{4-2(5)}{3} = \frac{4-10}{3} = -2$$

Hence, $x = -2$ and $y = 5$.

71. 2 man and 7 boys can do a piece of work in 4 days. It is done by 4 men and 4 boys in 3 days. How long would it take for one man or one boy to do it ?

Ans : [Board Term-1 2013]

Let the man can finish the work in x days and the boy can finish work in y days.

Work done by one man in one day $= \frac{1}{x}$

And work done by one boy in one day $= \frac{1}{y}$

$$\frac{2}{x} + \frac{7}{y} = \frac{1}{4} \quad ...(1)$$

and $$\frac{4}{x} + \frac{4}{y} = \frac{1}{3} \quad ...(2)$$

Let $\frac{1}{x}$ be a and $\frac{1}{y}$ be b, then we have

$$2a + 7b = \frac{1}{4} \quad ...(3)$$

and $$4a + 4b = \frac{1}{3} \quad ...(4)$$

Multiplying equation (3) by 2 and subtract equation (4) from it

$$10b = \frac{1}{6}$$

$$b = \frac{1}{60} = \frac{1}{y}$$

Thus $y = 60$ days.

Substituting $b = \frac{1}{60}$ in equation (3), we have

$$2a + \frac{7}{60} = \frac{1}{4}$$

$$2a = \frac{1}{4} - \frac{7}{60}$$

$$a = \frac{1}{15}$$

Now $\frac{1}{15} = \frac{1}{x}$

Thus $x = 15$ days.

72. In an election contested between A and B, A obtained votes equal to twice the no. of persons on the electoral roll who did not cast their votes and this later number was equal to twice his majority over B. If there were 1,8000 persons on the electoral roll. How many votes for B.

Ans : [Board Term-1 2012, Set-56]

Let x and y be the no. of votes for A and B respectively.
The no. of persons who did not vote is $18000 - x - y$.

We have $x = 2(18000 - x - y)$

$$3x + 2y = 36000 \quad ...(1)$$

and $(18000 - x - y) = 2(x - y)$

or $$3x - y = 18000 \quad ...(2)$$

Subtracting equation (2) from equation (1),

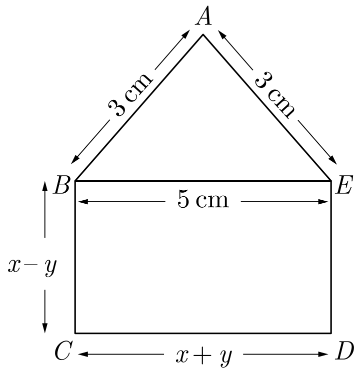
$$3y = 18000$$

$$y = 6000$$

Hence vote for B is 6000.

73. In the figure below $ABCDE$ is a pentagon with $BE || CD$ and $BC || DE$. BC is perpendicular to DC.

If the perimeter of $ABCDE$ is 21 cm, find the values of x and y.



Ans : [Board Term-1 2011]

Since $BC || DE$ and $BE || CD$ with $BC \perp DC$, $BCDE$ is a rectangle.

$$BE = CD,$$

$$x + y = 5 \quad ...(1)$$

and $$DE = BE = x - y$$

Since perimeter of $ABCDE$ is 21,

$$AB + BC + CD + DE + EA = 21$$

$$3 + x - y + x + y + x - y + 3 = 21$$

$$6 + 3x - y = 21$$

$$3x - y = 15$$

Adding equations (1) and (2), we get

$$4x = 20 \quad ...(2)$$

$$x = 5$$

Substituting the value of x in (1), we get

$$y = 0$$

Thus $x = 5$ and $y = 0$.

74. Solve for x and y :

$$\frac{x+1}{2} + \frac{y-1}{3} = 9 \; ; \; \frac{x-1}{3} + \frac{y+1}{2} = 8.$$

Ans : [Board Term-1 2011, Set-52]

We have $$\frac{x+1}{2} + \frac{y-1}{3} = 9$$

$$3(x+1) + 2(y-1) = 54$$

$$3x + 3 + 2y - 2 = 54$$

$$3x + 2y = 53 \quad (1)$$

and $$\frac{x-1}{3} + \frac{y+1}{2} = 8$$

$$2(x-1) + 3(y+1) = 48$$

$$2x - 2 + 3y + 3 = 48$$

$$2x + 3y = 47 \quad (2)$$

Multiplying equation (1) by 3 we have

$$9x + 6y = 159 \quad (3)$$

Multiplying equation (2) by 2 we have

$$4x + 6y = 94 \quad (4)$$

Subtracting equation (4) from (3) we have

$$5x = 65$$

or $$x = 13$$

Substitute the value of x in equation (2),

$$2(13) + 3y = 47$$

$$3y = 47 - 26 = 21$$

$$y = \frac{21}{3} = 7$$

Hence, $x = 13$ and $y = 7$

75. Solve for x and y :

$$\frac{6}{x-1} - \frac{3}{y-2} = 1$$

$$\frac{5}{x-1} - \frac{1}{y-2} = 2, \text{ where } x \neq 1,\ y \neq 2.$$

Ans : [Board Term-1 2011]

We have $$\frac{6}{x-1} - \frac{3}{y-2} = 1 \quad (1)$$

$$\frac{5}{x-1} - \frac{1}{y-2} = 2, \quad (2)$$

Let $\frac{1}{x-1} = p$ and $\frac{1}{y-2} = q$. then given equations become

$$6p - 3q = 1 \quad ...(3)$$

and $$5p - q = 2 \quad ...(4)$$

Multiplying equation (4) by 3 and adding in equation (3), we have

$$21p = 7$$

$$p = \frac{7}{21} = \frac{1}{3}$$

Substituting this value of p in equation (3), we have

$$6\left(\frac{1}{3}\right) - 3q = 1$$

$$2 - 3q = 1 \Rightarrow q = \frac{1}{3}$$

Now, $\frac{1}{x-1} = p = \frac{1}{3}$

or, $x - 1 = 3 \Rightarrow x = 4$

and $\frac{1}{y-2} = q = \frac{1}{3}$

or, $y - 2 = 3 \Rightarrow y = 5$

Hence $x = 4$ and, $y = 5$.

76. Seven times a two digit number is equal to four times the number obtained by reversing the order of its digits. If the difference of the digits is 3, determine the number.

Ans : [Board Term-1 2017]

Let the ten's and unit digit by y and x respectively,
So the number is $10y + x$
The number when digits are reversed becomes $10x + y$

Thus $7(10y + x) = 4(10x + y)$

$$70y + 7x = 40x + 4y$$

$$70y - 4y = 40x - 7x$$

$$2y = x \quad ...(1)$$

or $x - y = 3 \quad ...(2)$

From (1) and (2) we get

$$y = 3 \text{ and } x = 6$$

Hence the number is 36.

77. Solve the following pair of equations for x and y :

$$\frac{a^2}{x} - \frac{b^2}{y} = 0, \frac{a^2 b}{x} + \frac{b^2 a}{y} = a + b, \qquad x \neq 0; y \neq 0.$$

Ans : [Board Term-1 2011]

We have $\frac{a^2}{x} - \frac{b^2}{y} = 0$

$$\frac{a^2 b}{x} + \frac{b^2 a}{y} = a + b = a + b$$

Substituting $p = \frac{1}{x}$ and $q = \frac{1}{y}$ in the given equations,

$$a^2 p - b^2 q = 0 \quad ...(1)$$

$$a^2 bp + b^2 aq = a + b \quad ...(2)$$

Multiplying equation (1), by a

$$a^3 p - b^2 aq = 0 \quad ...(3)$$

Adding equation (2) and equation (3),

$$(a^3 + a^2 b)p = a + b$$

or, $p = \frac{(a+b)}{a^2(a+b)} = \frac{1}{a^2}$

Substituting the value of p in equation (1),

$$a^2\left(\frac{1}{a^2}\right) - b^2 q = 0 \Rightarrow q = \frac{1}{b^2}$$

Now, $p = \frac{1}{x} = \frac{1}{a^2} \Rightarrow x = a^2$

and $q = \frac{1}{y} = \frac{1}{b^2} \Rightarrow y = b^2$

Hence, $x = a^2$ and $y = b^2$

78. Solve for x and y :

$$ax + by = \frac{a+b}{2}$$

$$3x + 5y = 4$$

Ans : [Board Term-1 2011, Set-44]

We have $ax + by = \frac{a+b}{2}$

or $2ax + 2by = a + b \quad ...(1)$

and $3x + 5y = 4 \quad ...(2)$

Multiplying equation (1) by 5 we have

$$10ax + 10by = 5a + 5b \quad ...(3)$$

Multiplying equation (2) by $2b$, we have

$$6bx + 10by = 8b \quad ...(4)$$

Subtracting (4) from (3) we have

$$(10a - 6b)x = 5a - 3b$$

or $x = \frac{5a - 3b}{10a - 6b} = \frac{1}{2}$

Substitute $x = \frac{1}{2}$ in equation (2), we get

$$3 \times \frac{1}{2} + 5y = 4$$

$$5y = 4 - \frac{3}{2} = \frac{5}{2}$$

$$y = \frac{5}{2 \times 5} = \frac{1}{2}$$

Hence $x = \frac{1}{2}$ and $y = \frac{1}{2}$.

79. Solve the following pair of equations for x and y :

$$4x+\frac{6}{y}=15, 6x-\frac{8}{y}=14$$

and also find the value of p such that $y=px-2$.

Ans : [Board Term-1 2011, Set-60]

We have
$$4x+\frac{6}{y}=15 \quad (1)$$
$$6x-\frac{8}{y}=14, \quad (2)$$

Let $\frac{1}{y}=z$, the given equations become
$$4x+6z=15 \quad ...(3)$$
$$6x-8z=14 \quad ...(4)$$

Multiply equation (3) by 4 we have
$$16x+24z=60 \quad (5)$$

Multiply equation (4) by 3 we have
$$18x-24z=24 \quad (6)$$

Adding equation (5) and (6) we have
$$34x=102$$
$$x=\frac{102}{34}=3$$

Substitute the value of x in equation (3),
$$4(3)+6z=15$$
$$6z=15-12=3$$
$$z=\frac{3}{6}=\frac{1}{2}$$

Now
$$z=\frac{1}{y}=\frac{1}{2} \Rightarrow y=2$$

Hence $x=3$ and $y=2$.

Again
$$y=px-2$$
$$2=p(3)-2$$
$$3p=4$$

Thus
$$p=\frac{4}{3}$$

80. A chemist has one solution which is 50 % acid and a second which is 25 % acid. How much of each should be mixed to make 10 litre of 40 % acid solution.

Ans : [Board Term-1 2015, JRTSY]

Let 50 % acids in the solution be x and 25 % of other solution be y.

Total volume in the mixture
$$x+y=10 \quad ...(1)\ 1$$

and
$$\frac{50}{100}x+\frac{25}{100}y=\frac{40}{100}\times 10$$
$$2x+y=16 \quad ...(2)\ 1$$

Subtracting equation (1) from (2) we have
$$x=6$$

Substituting this value of x in equation (1) we get
$$6+y=10$$
$$y=4$$

Hence, $x=6$ and $y=4$.

81. Find whether the following pair of linear equations has a unique solutions. If yes, find the solution :
$$7x-4y=49, 5x-6y=57.$$

Ans : [Board Term-1 2011]

We have
$$7x-4y=49 \quad (1)$$
$$5x-6y=57 \quad (2)$$

Comparing with the equation $a_1x+b_1y=c_1$,
$$a_1=7, b_1=-4, c_1=49$$
$$a_2=5, b_2=-6, c_2=57$$

Since,
$$\frac{a_1}{a_2}=\frac{7}{5} \text{ and } \frac{b_1}{b_2}=\frac{4}{6}$$
$$\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$$

So, system has a unique solution.

Multiply equation (1) by 5 we get
$$35x-20y=245 \quad (3)$$

Multiply equation (2) by 7 we get
$$35x-42y=399 \quad (4)$$

Subtracting (4) by (3) we have
$$22y=-154$$
$$y=-7$$

Putting the value of y in equation (2),
$$5x-6(-7)=57$$
$$5x=57-42=15$$
$$x=3$$

Hence $x=3$ and $y=-7$

FOUR MARKS QUESTIONS

82. Determine graphically the coordinates of the vertices of triangle, the equations of whose sides are given by $2y - x = 8$, $5y - x = 14$ and $y - 2x = 1$.

Ans : [Board 2020 Delhi Standard]

We have $2y - x = 8$

L_1 : $x = 2y - 8$

y	0	4	5
$x = 2y - 8$	-8	0	2

$5y - x = 14$

L_2 : $x = 5y - 14$

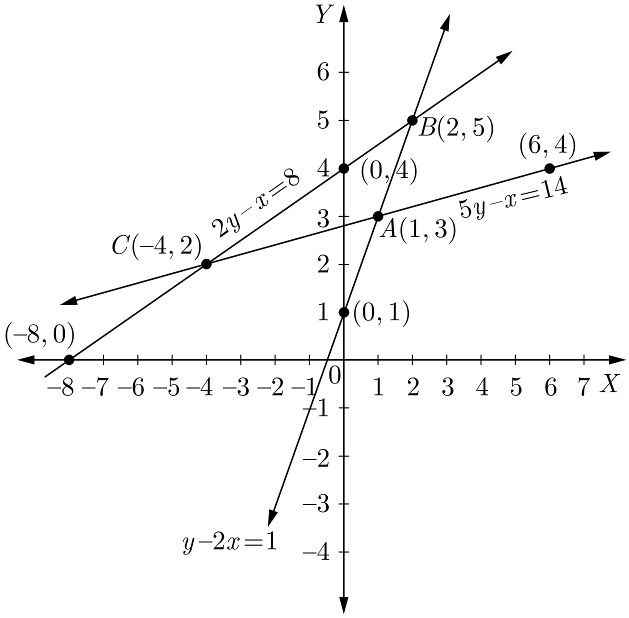
y	3	4	2
$x = 5y - 14$	1	6	-4

and $y - 2x = 1$

L_3 : $y = 1 + 2x$

x	0	1	2
$y = 1 + 2x$	1	3	5

Plotting the above points and drawing lines joining them, we get the graphical representation:



Hence, the coordinates of the vertices of a triangle ABC are $A(1,3)$, $B(2,5)$ and $C(-4,2)$.

83. A man can row a boat downstream 20 km in 2 hours and upstream 4 km in 2 hours. Find his speed of rowing in still water. Also find the speed of the stream.

Ans : [Board 2020 Delhi Standard]

Let x be the speed of the boat in still water and y be the speed of the stream.

Relative Speed of boat in upstream will be $(x - y)$ and relative speed of boat in downstream will be $(x + y)$.

According to question, we have

$$\frac{20}{x+y} = 2$$

$$x + y = 10 \quad ...(1)$$

and
$$\frac{4}{x-y} = 2$$

$$x - y = 2 \quad ...(2)$$

Adding equation (1) and (2), we have

$$2x = 12 \Rightarrow x = 6 \text{ km/hr}$$

Substituting the value of x is equation (1) we have,

$$6 + y = 10 \Rightarrow y = 10 - 6 = 4 \text{ km/hr}$$

Thus speed of a boat in still water is 6 km/hr and speed of the stream 4 km/hr.

84. It can take 12 hours to fill a swimming pool using two pipes. If the pipe of larger diameter is used for four hours and the pipe of smaller diameter for 9 hours, only half of the pool can be filled. How long would it take for each pipe to fill the pool separately?

Ans : [Board 2020 OD Standard]

Let x be time taken to fill the pool by the larger diameter pipe and y be the time taken to fill the pool by the smaller diameter pipe.

According to question,

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{12} \quad ...(1)$$

and
$$\frac{4}{x} + \frac{9}{y} = \frac{1}{2} \quad ...(2)$$

Multiplying equation (1) by 9 and subtracting from equation (2), we get

$$\frac{5}{x} = \frac{9}{12} - \frac{1}{2} = \frac{1}{4}$$

$$x = 20$$

Substituting the value of x in equation (1), we have

$$\frac{1}{20}+\frac{1}{y} = \frac{1}{12}$$

$$\frac{1}{y} = \frac{1}{12}-\frac{1}{20} = \frac{5-3}{60}$$

$$\frac{1}{y} = \frac{2}{60} = \frac{1}{30} \Rightarrow y = 30$$

Hence, time taken to fill the pool by the larger and smaller diameter pipe are 20 hrs and 30 hrs respectively.

85. For what value of k, which the following pair of linear equations have infinitely many solutions:
$2x+3y=7$ and $(k+1)x+(2k-1)y=4k+1$

Ans : [Board 2019 Delhi Standard]

We have $$2x+3y = 7$$

and $$(k+1)x+(2k-1)y = 4k+1$$

Here $$\frac{a_1}{a_2} = \frac{2}{k+1}, \frac{b_1}{b_2} = \frac{3}{(2k-1)}$$

and $$\frac{c_1}{c_2} = \frac{-7}{-(4k+1)} = \frac{7}{(4k+1)}$$

For infinite many solutions

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

For $\frac{a_1}{a_2} = \frac{c_1}{c_2}$ we have

$$\frac{2}{k+1} = \frac{7}{4k+1}$$

$$2(4k+1) = 7(k+1)$$

$$8k+2 = 7k+7$$

$$k = 5$$

Hence, the value of k is 5, for which the given equation have infinitely many solutions.

86. Find c if the system of equations $cx+3y+(3-c)=0; 12x+cy-c=0$ has infinitely many solutions?

Ans : [Board 2019 Delhi]

We have $$cx+3y+(3-c) = 0$$

$$12x+cy-c = 0$$

Here, $$\frac{a_1}{a_2} = \frac{c}{12}, \frac{b_1}{b_2} = \frac{3}{c}, \frac{c_1}{c_2} = \frac{3-c}{-c}$$

For infinite many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

For $\frac{a_1}{a_2} = \frac{c_1}{c_2}$ we have,

$$\frac{c}{12} = \frac{3-c}{-c}$$

$$-c^2 = 36-12c$$

$$-c^2+12c-36 = 0$$

$$c^2-12c+36 = 0$$

$$c^2-6c-6c+36 = 0$$

$$c(c-6)-6(c-6) = 0$$

$$(c-6)(c-6) = 0 \Rightarrow c = 6$$

and for $\frac{b_1}{b_2} = \frac{c_1}{c_2}$,

$$\frac{3}{c} = \frac{3-c}{-c}$$

$$-3c = 3c-c^2$$

$$c^2-6c = 0$$

$$c(c-6) = 0 \Rightarrow c = 6 \text{ or } c \neq 0$$

Hence, the value of c is 6, for which the given equations have infinitely many solutions.

87. A father's age is three times the sum of the ages of his two children. After 5 years his age will be two times the sum of their ages. Find the present age of the father.

Ans : [Board 2019 Delhi]

Let x be the age of father and y be the sum of the ages of his children.

After 5 years,

Father's age $= (x+5)$ years

Sum of ages of his children $= (y+10)$ years

According to the given condition,

$$x = 3y \quad ...(1)$$

and $$x+5 = 2(y+10)$$

or, $$x-2y = 15 \quad ...(2)$$

Solving equation (1) and (2), we have

$$3y - 2y = 15 \Rightarrow y = 15$$

Substituting value of y in equation (1), we get

$$x = 3 \times 15 = 45$$

Hence, father's present age is 45,

88. Two water taps together can fill a tank in $1\frac{7}{8}$ hours. The tap with longer diameter takes 2 hours less than the tap with smaller one to fill the tank separately. Find the time in which each tap can fill the tank separately.

Ans : [Board 2019 Delhi]

Let t be the time taken by the smaller diameter top. Time for larger tap diameter will be $t-2$.

Total time taken $= 1\frac{7}{8} = \frac{15}{8}h$.

Portion filled in one hour by smaller diameter tap will $\frac{1}{t}$ and by lager diameter tap will be $\frac{1}{t-2}$

According to the problem,

$$\frac{1}{t} + \frac{1}{t-2} = \frac{8}{15}$$

$$\frac{t-2+t}{t(t-2)} = \frac{8}{15}$$

$$15(2t-2) = 8t(t-2)$$

$$30t - 30 = 8t^2 - 16t$$

$$8t^2 - 46t + 30 = 0$$

$$4t^2 - 23t + 15 = 0$$

$$4t^2 - 20t - 3t + 30 = 0$$

$$(4t-3)(t-5) = 0 \Rightarrow t = \frac{3}{4} \text{ or } t = 5$$

If $t = \frac{3}{4}$ then $t - 2 = \frac{3}{4} - 2 = \frac{-5}{4}$

Since, time cannot be negative, we neglect $t = \frac{3}{4}$

Therefore, $t = 5$

and $t - 2 = 5 - 2 = 3$

Hence, time taken by larger tap is 3 hours and time taken by smaller is 5 hours

89. A boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours, it can go 40 km upstream and 55 km downstream. Determine the speed of the stream and that of the boat in still water.

Ans : [Board 2019 Delhi]

Let x be the speed of boat in still water and y be the speed of stream.

Relative speed of boat in downstream will be $x+y$ and relative speed of boat in upstream will be $x-y$.

Time taken to go 30 km upstream,

$$t_1 = \frac{30}{x-y}$$

Time taken to go 44 km downstream,

$$t_2 = \frac{40}{x+y}$$

According to the first condition we have

$$\frac{30}{x-y} + \frac{44}{x+y} = 10 \quad ...(1)$$

Similarly according to the second condition we have

$$\frac{40}{x-y} + \frac{55}{x+y} = 13 \quad ...(2)$$

Let $\frac{1}{x-y} = u$ and $\frac{1}{x+y} = \upsilon$, then we have

$$30u + 44\upsilon = 10 \quad ...(3)$$

$$40u + 55\upsilon = 13 \quad ...(4)$$

Multiplying equation (3) by 4 and equation (4) by 3 and then subtracting we have

$$11v = 1 \Rightarrow \upsilon = \frac{1}{11}$$

Multiplying equation (3) by 5 and equation (4) by 4 and then subtracting we have

$$-10u = -2 \quad ...(4)$$

$$u = \frac{1}{5}$$

Now $u = \frac{1}{x-y} = \frac{1}{5}$

$$x - y = 5 \quad (5)$$

and $\upsilon = \frac{1}{x+y} = \frac{1}{11}$

$$x + y = 11 \quad (6)$$

Adding equation (5) and (6), we get

$$2x = 16 \Rightarrow x = 8$$

Substitute value of x in equation (5), we get

$$8 - y = 5 \Rightarrow y = 3$$

Hence speed of boat in still water is 8 km/hour and and speed of stream is 3 km/hour.

90. Sumit is 3 times as old as his son. Five years later he shall be two and a half times as old as his son. How old is Sumit at present?

Ans : [Board 2019 OD]

Let x be Sumit's present age and y be his son's present age.

According to given condition,

$$x = 3y$$

After five years, ...(1)

$$\text{Sumit's age} = x+5$$

and $$\text{His son's age} = y+5$$

Now, again according to given condition,

$$x+5 = 2\frac{1}{2}(y+5)$$

$$x+5 = \frac{5}{2}(y+5)$$

$$2(x+5) = 5(y+5)$$

$$2x+10 = 5y+25$$

$$2x = 5y+15$$

$$2(3y) = 5y+15 \quad \text{[from eq (1)]}$$

$$6y = 5y+15$$

$$y = 15$$

Again, from eq (1)

$$x = 3y = 3\times 15 = 45$$

Hence, Sumit's present age is 45 years.

91. For what value of k, will the following pair of equations have infinitely many solutions:
$2x+3y=7$ and $(k+2)x-3(1-k)y=5k+1$

Ans : [Board 2019 OD]

We have $$2x+3y = 7 \quad ...(1)$$

and $$(k+2)x-3(1-k)y = 5k+1 \quad ...(2)$$

Comparing equation (1) with $a_1x+b_1y=c_1$ and equation (2) by $a_2x+b_2y=c_2$ we have

$$a_1 = 2,\ b_1 = 3,\ c_1 = 7$$

and $$a_2 = (k+2),\ b_2 = -3(1-k),\ c_1 = 5k+1$$

Here, $$\frac{a_1}{a_2} = \frac{2}{k+2},$$

$$\frac{b_1}{b_2} = \frac{3}{-3(1-k)},\ \frac{c_1}{c_2} = \frac{7}{5k+1}$$

For a pair of linear equations to have infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

So, $$\frac{2}{k+2} = \frac{3}{-3(1-k)} = \frac{7}{5k+1}$$

$$\frac{2}{k+2} = \frac{3}{-3(1-k)}$$

$$2(1-k) = -(k+2)$$

$$2-2k = -k-2 \Rightarrow k = 4$$

Hence, for $k=4$, the pair of linear equations has infinitely many solutions.

92. The total cost of a certain length of a piece of cloth is ₹200. If the piece was 5 m longer and each metre of cloth costs ₹2 less, the cost of the piece would have remained unchanged. How long is the piece and what is its original rate per metre?

Ans : [Board 2019 OD]

Let x be the length of the cloth and y be the cost of cloth per meter.

Now $$x\times y = 200$$

$$y = \frac{200}{x} \quad ...(1)$$

According to given conditions,

1. If the piece were 5 m longer
2. Each meter of cloth costed ₹ 2 less

i.e., $$(x+5)(y-2) = 200$$

$$xy-2x+5y-10 = 200$$

$$xy-2x+5y = 210$$

$$x\left(\frac{200}{x}\right)-2x+5\left(\frac{200}{x}\right) = 210$$

$$200-2x+\frac{1000}{x} = 210$$

$$\frac{1000}{x}-2x = 10$$

$$1000-2x^2 = 10x$$

$$x^2+25x-20x-500 = 0$$

$$x(x+25)-20(x+25) = 0$$

$$(x+25)(x-20) = 0$$

$$x = -25,\ 20$$

Neglecting $x = -25$ we get $x = 20$.

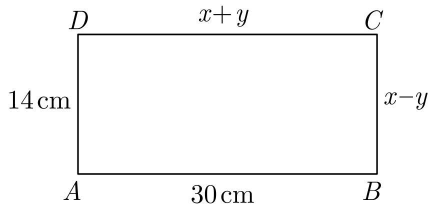
Now from equation (1), we have

$$y = \frac{200}{x} = \frac{200}{20} = 10$$

Hence, length of the piece of cloths is 20 m and rate per meter is ₹ 10.

93. In Figure, $ABCD$ is a rectangle. Find the values of

x and y.



Ans : [Board 2018]

Since $ABCD$ is a rectangle, we have

$$AB = CD \text{ and } BC = AD$$

Now $$x + y = 30 \quad ...(1)$$

$$x - y = 14 \quad ...(2)$$

Adding equation (1) and (3) we obtain,

$$2x = 44 \Rightarrow x = \frac{44}{2} = 22$$

Substituting value of x in equation (1) we have

$$22 + y = 30$$

$$y = 30 - 22 = 8$$

$$x = 22\,\text{cm and } y = 8\,\text{cm}$$

94. For Uttarakhand flood victims two sections A and B of class contributed Rs. 1,500. If the contribution of X-A was Rs. 100 less than that of X-B, find graphically the amounts contributed by both the sections.

Ans : [Board Term-1 2016]

Let amount contributed by two sections X-A and X-B be Rs. x and Rs.y.

$$x + y = 1,500 \quad ...(1)$$

$$y - x = 100 \quad ...(2)$$

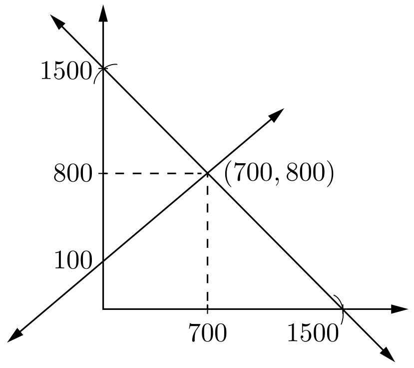
From (1) $y = 1500 - x$

x	0	700	1,500
y	1,500	800	0

From (2) $y = 100 + x$

x	0	700
y	100	800

Plotting the above points and drawing lines joining them, we get the following graph.



Clearly, the two lines intersect at point (700, 800)

Hence X-A contributes 700 Rs and X-B contributes 800 Rs.

95. Determine graphically whether the following pair of linear equations :

$$3x - y = 7$$

$$2x + 5y + 1 = 0 \text{ has :}$$

a. unique solution
b. infinitely many solutions or
c. no solution.

Ans : [Board Term-1 2015]

We have $$3x - y = 7$$

or $$3x - y - 7 = 0 \quad (1)$$

Here $a_1 = 3, b_1 = 1, c_1 = -7$

$$2x + 5y + 1 = 0 \quad (2)$$

Here $a_2 = 2, b_2 = 5, c_2 = 1$

Now $$\frac{a_1}{a_2} = \frac{3}{2}, \frac{b_1}{b_2} = \frac{-1}{5}$$

Since $\frac{3}{2} \neq \frac{-1}{5}$, thus $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence, given pair of linear equations has a unique solution.

Now line (1) $y = 3x - 7$

x	0	2	3
y	−7	−1	2

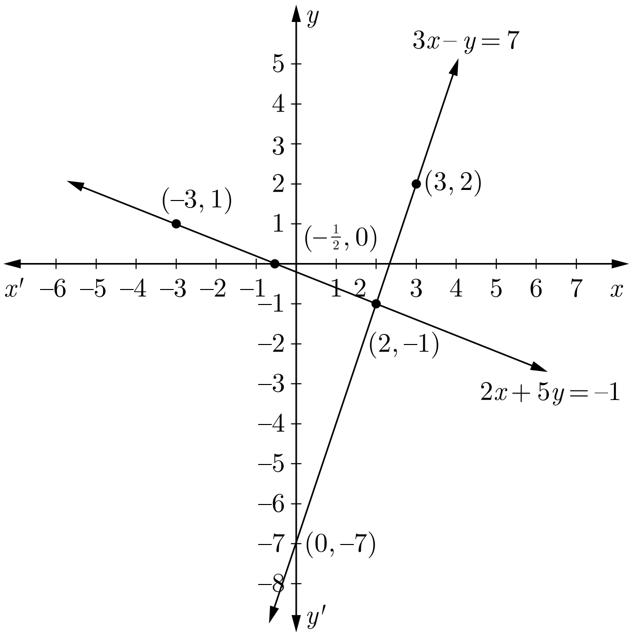
and line (2)

$$2x + 5y + 1 = 0$$

or, $$y = \frac{-1 - 2x}{5}$$

x	2	-3
y	-1	1

Plotting the above points and drawing lines joining them, we get the following graph.



Clearly, the two lines intersect at point $(2, -1)$.
Hence $x = 2$ and $y = -1$

96. Draw the graphs of the pair of linear equations :
$x + 2y = 5$ and $2x - 3y = -4$

Also find the points where the lines meet the $x-$axis.

Ans : [Board Term-1 2015]

We have $$x + 2y = 5$$

or, $$y = \frac{5 - x}{2}$$

c123

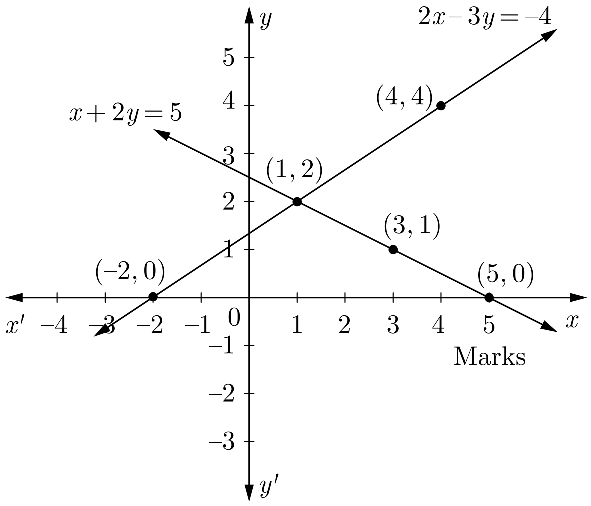
x	1	3	5
y	2	1	0

and $$2x - 3y = -4$$

or, $$y = \frac{2x + 4}{3}$$

x	1	4	-2
y	2	4	0

Plotting the above points and drawing lines joining them, we get the following graph.



Clearly two lines meet x-axis at $(5, 0)$ and $(-2, 0)$ respectively.

97. Solve graphically the pair of linear equations :
$3x - 4y + 3 = 0$ and $3x + 4y - 21 = 0$

Find the co-ordinates of the vertices of the triangular region formed by these lines and $x-$axis. Also, calculate the area of this triangle.

Ans : [Board Term-1 2015]

We have $$3x - 4y + 3 = 0$$

or, $$y = \frac{3x + 3}{4}$$

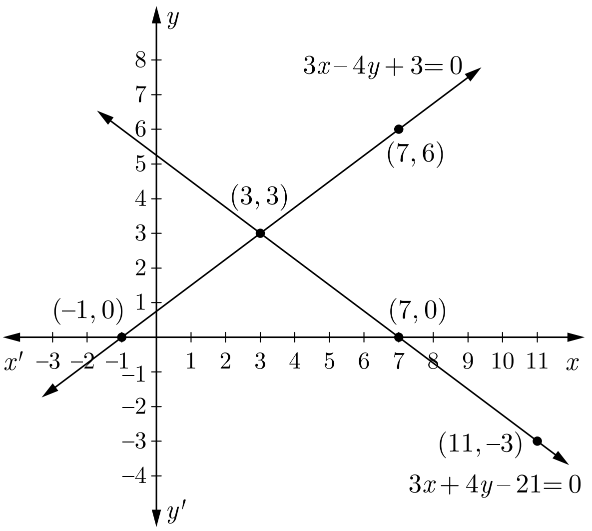
x	3	7	-1
y	3	6	0

and $$3x + 4y - 21 = 0$$

or, $$y = \frac{21 - 3x}{4}$$

x	3	7	11
y	3	0	-2

Plotting the above points and drawing lines joining them, we get the following graph.



Clearly, the two lines intersect at point $(3,3)$.

(a) These lines intersect each other at point$(3,3)$. Hence $x=3$ and $y=3$

(b) The vertices of triangular region are $(3,3)$, $(-1,0)$ and $(7,0)$.

(c) Area of $\Delta = \frac{1}{2}\times 8\times 3 = 12$

Hence, Area of obtained Δ is 12 sq unit.

98. Aftab tells his daughter, '7 years ago, I was seven times as old as you were then. Also, 3 years from now, I shall be three times as old as you will be.' Represent this situation algebraically and graphically.

Ans : [Board Term-1 2015, NCERT]

Let the present age of Aftab be x years and the age of daughter be y years.

7 years ago father's(Aftab) age $=(x-7)$years

7 years ago daughter's age $=(y-7$ years

According to the question,

$$(x-7) = 7(y-7)$$

or, $$(x-7y) = -42 \qquad (1)$$

After 3 years father's(Aftab) age $=(x+3)$ years

After 3 years daughter's age $=(y+3)$ years

According to the condition,

$$x+3 = 3(y+3)$$

or, $$x-3y = 6 \qquad (2)$$

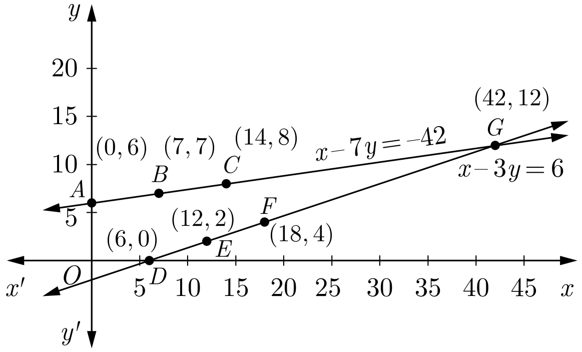
From equation(1) $x-7y = -42$

x	0	7	14
$y=\frac{x+42}{7}$	6	7	8

From equation (2) $x-3y = 6$

x	6	12	18
$y=\frac{x-6}{3}$	0	2	4

Plotting the above points and drawing lines joining them, we get the following graph.



Two lines obtained intersect each other at $(42,12)$

Hence, father's age $= 42$ years

and daughter's age $= 12$ years

99. The cost of 2 kg of apples and 1kg of grapes on a day was found to be Rs. 160. After a month, the cost of 4kg of apples and 2kg of grapes is Rs. 300. Represent the situations algebraically and geometrically.

Ans : [Board Term-1 2013, Set DDE-E, NCERT]

Let the cost of 1 kg of apples be Rs.x and cost of 1 kg of grapes be Rs. y.

The given conditions can be represented given by the following equations :

$$2x+y = 160 \qquad ...(1)$$

$$4x+2y = 300 \qquad ...(2)$$

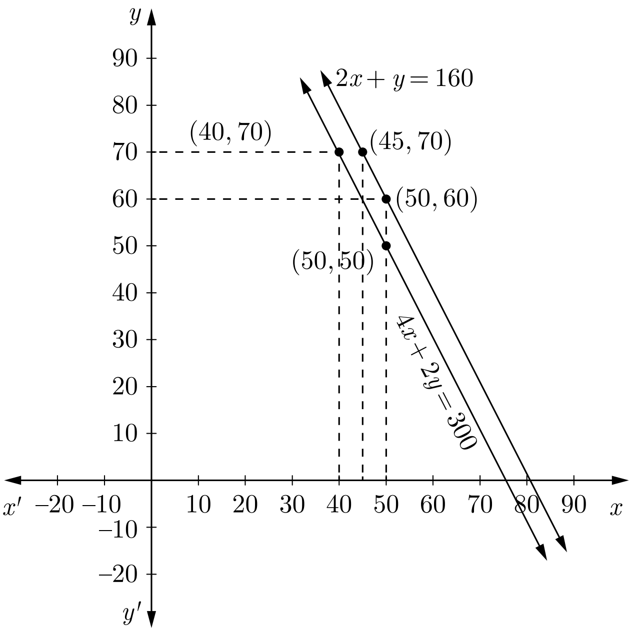
From equation (1) $y = 160-2x$

x	50	45
y	60	70

From equation (2) $y = 150-2x$

x	50	40
y	50	70

Plotting these points on graph, we get two parallel line as shown below.



100. Draw the graphs of the equations $x-y+1=0$ and $3x+2y-12=0$. Determine the co-ordinates of the vertices of the triangle formed by these lines and the X-axis and shade the triangular region.

Ans : [Board Term-1 2013 NCERT]

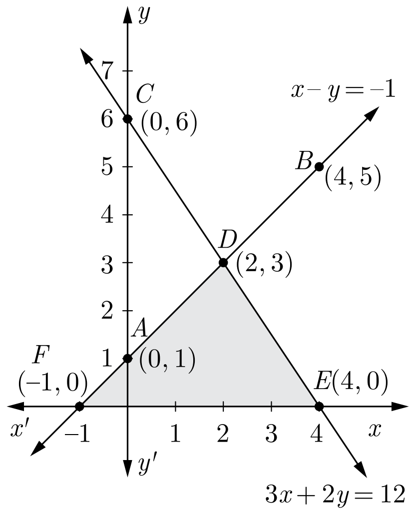
We have $x-y+1=0$ ...(1)

x	0	4	2
$y=x+1$	1	5	3

and $3x+2y-12=0$ ...(2)

x	0	2	4
$y=\frac{12-3x}{2}$	6	3	0

Plotting the above points and drawing lines joining them, we get the following graph.



Clearly, the two lines intersect at point $D(2,3)$.

Hence, $x=2$ and $y=3$ is the solution of the given pair of equations. The line CD intersects the x-axis at the point $E(4,0)$ and the line AB intersects the x-axis at the points $F(-1,0)$. Hence, the co-ordinates of the vertices of the triangle are $D(2,3)$, $E(4,0)$ and $F(-1,0)$.

101. Solve the following pair of linear equations graphically:

$2x+3y=12$ and $x-y=1$

Find the area of the region bounded by the two lines representing the above equations and y-axis.

Ans : [Board Term-1 2012, Set-58]

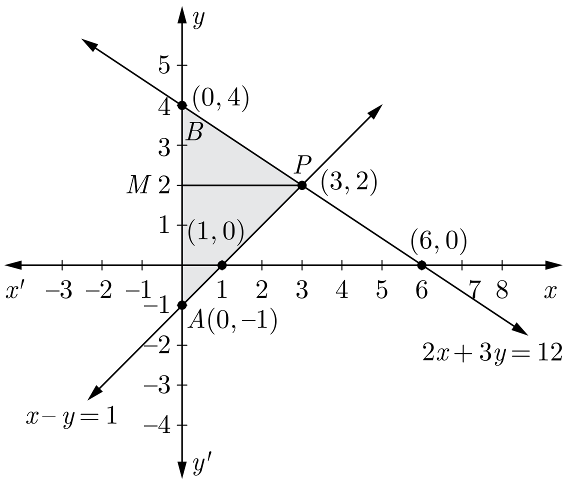
We have $2x+3y=12 \Rightarrow y=\frac{12-2x}{3}$

x	0	6	3
y	4	0	2

We have $x-y=1 \Rightarrow y=x-1$

x	0	1	3
y	1	0	2

Plotting the above points and drawing lines joining them, we get the following graph.



Clearly, the two lines intersect at point $p(3,2)$.

Hence, $x = 3$ and $y = 2$

Area of shaded triangle region,

$$\text{Area of } \Delta PAB = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times AB \times PM$$

$$= \frac{1}{2} \times 5 \times 3$$

$$= 7.5 \text{ square unit.}$$

102. Solve the following pair of linear equations graphically:

$x + 3y = 12,\ 2x - 3y = 12$

Also shade the region bounded by the line $2x - 3y = 2$ and both the co-ordinate axes.

Ans : [Board Term-1 2013 FFC, 2012, Set-35, 48]

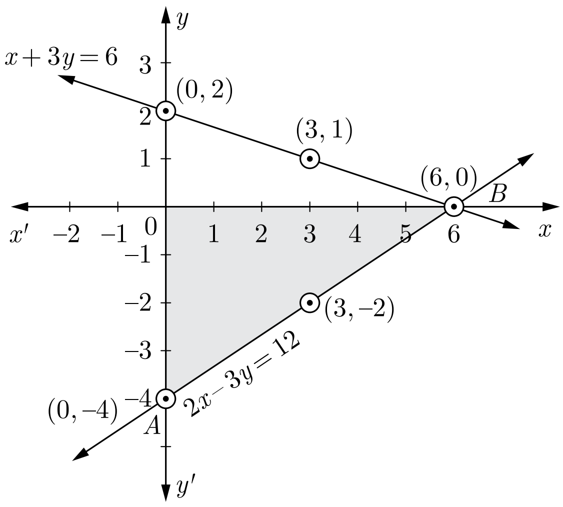
We have $\quad x + 3y = 6 \Rightarrow y = \dfrac{6 - x}{3}$ ...(1)

x	3	6	0
y	1	0	2

and $\quad 2x - 3y = 12 \Rightarrow y = \dfrac{2x - 12}{3}$

x	0	6	3
y	-4	0	-2

Plotting the above points and drawing lines joining them, we get the following graph.



The two lines intersect each other at point $B(6,0)$.

Hence, $x = 6$ and $y = 0$

Again ΔOAB is the region bounded by the line $2x - 3y = 12$ and both the co-ordinate axes.

103. Solve the following pair of linear equations graphically:

$x - y = 1,\ 2x + y = 8$

Also find the co-ordinates of the points where the lines represented by the above equation intersect $y-axis$.

Ans : [Board Term-1 2012, Set-56]

We have $\quad x - y = 1 \Rightarrow y = x - 1$

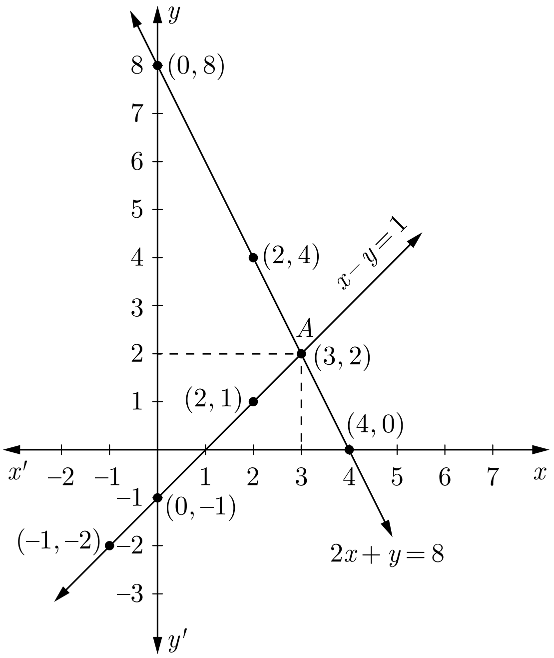
x	2	3	-1

y	1	2	-2

and $\qquad 2x+y=8 \Rightarrow y=8-2x$

x	2	4	0
y	4	0	8

Plotting the above points and drawing lines joining them, we get the following graph.



The two lines intersect each other at point $A(3,2)$. Thus solution of given equations is $x=3, y=2$.

Again, $x-y=1$ intersects $y-axis$ at $(0,-1)$

and $\quad 2x+y=8\,y-axis$ at $(0,8)$.

104. Draw the graph of the following equations:

$$2x-y=1,\ x+2y=13$$

Find the solution of the equations from the graph and shade the triangular region formed by the lines and the y-axis.

Ans : [Board Term-1 2012 Set-52]

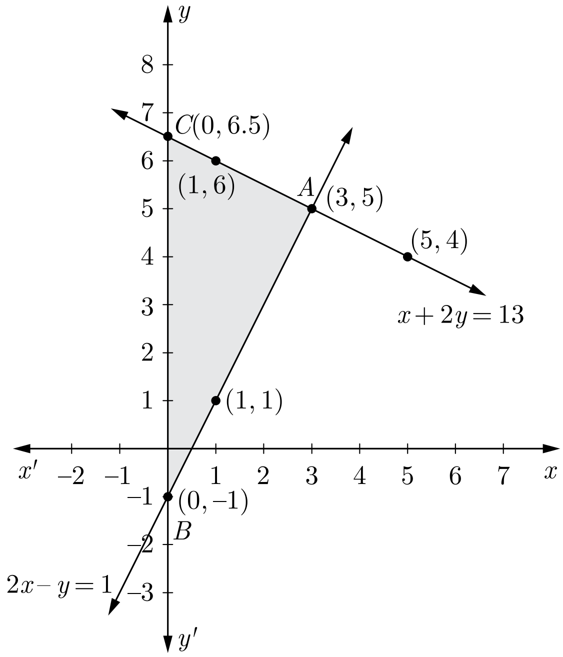
We have $\qquad 2x-y=1 \Rightarrow y=2x-1$

x	0	1	3
y	-1	1	5

and $\qquad x+2y=13 \Rightarrow y=\dfrac{13-x}{2}$

x	1	3	5
y	6	5	4

Plotting the above points and drawing lines joining them, we get the following graph.



Clearly two obtained lines intersect at point $A(3,5)$.

Hence, $x=3$ and $y=5$

ABC is the triangular shaded region formed by the obtained lines with the y-axis.

105. Solve the following pair of equations graphically:

$$2x+3y=12,\ x-y-1=0.$$

Shade the region between the two lines represented by the above equations and the X-axis.

Ans : [Board Term-1 2012, Set-48]

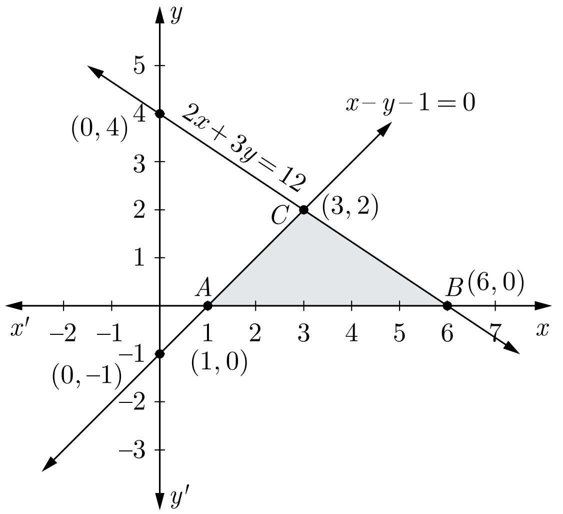
We have $\qquad 2x+3y=12 \Rightarrow y=\dfrac{12-2x}{3}$

x	0	6	3
y	4	0	2

also $\qquad x-y=1 \Rightarrow y=x-1$

x	0	1	3
y	-1	0	2

Plotting the above points and drawing lines joining them, we get the following graph.



The two lines intersect each other at point $(3,2)$,
Hence, $x = 3$ and $y = 2$.

ΔABC is the region between the two lines represented by the given equations and the X-axis.

106. 4 chairs and 3 tables cost Rs 2100 and 5 chairs and 2 tables cost Rs 1750. Find the cost of none chair and one table separately.

Ans : [Board Term-1 2015]

Let cost of 1 chair be Rs x and cost of 1 table be Rs y According to the question,

$$4x + 3y = 2100 \quad ...(1)$$

$$5x + 2y = 1750 \quad ...(2)$$

Multiplying equation (1) by 2 and equation (2) by 3,

$$8x + 6y = 4200 \quad ...(3)$$

$$15x + 6y = 5250 \quad ...(iv)$$

Subtracting equation (3) from (4) we have

$$7x = 1050$$

$$x = 150$$

Substituting the value of x in (1), $y = 500$

Thus cost of chair and table is Rs 150, Rs 500 respectively.

107. Solve the following pair of equations :

$$\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2 \text{ and } \frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1$$

Ans : [Board Term-1 2015]

We have $$\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2$$

$$\frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1$$

Substitute $\frac{1}{\sqrt{x}} = X$ and $\frac{1}{\sqrt{y}} = Y$

$$2X + 3Y = 2 \quad ...(1)$$

$$4X - 9Y = -1 \quad ...(2)$$

Multiplying equation (1) by 3, and adding in (2) we get

$$10X = 5 \Rightarrow X = \frac{5}{10} = \frac{1}{2}$$

Thus $$\frac{1}{\sqrt{x}} = \frac{1}{2} \Rightarrow x = 4$$

Putting the value of X in equation (1), we get

$$2 \times \frac{1}{2} + 3y = 2$$

$$3Y = 2 - 1$$

$$Y = \frac{1}{3}$$

Now $$Y = \frac{1}{3} \Rightarrow \frac{1}{\sqrt{y}} = \frac{1}{3} \Rightarrow y = 9$$

Hence $x = 4$, $y = 9$.

108. Solve for x and y :

$$2x - y + 3 = 0$$

$$3x - 5y + 1 = 0$$

Ans : [Board Term-1 2015]

We have $$2x - y + 3 = 0 \quad ...(1)$$

$$3x - 5y + 1 = 0 \quad ...(2)$$

Multiplying equation (1) by 5,and subtracting (2) from it we have

$$7x = -14$$

$$x = \frac{-14}{7} = -2$$

Substituting the value of x in equation (1) we get

$$2x - y + 3 = 0$$

$$2(-2) - y + 3 = 0$$

$$-4 - y + 3 = 0$$

$$-y - 1 = 0$$

$$y = -1$$

Hence, $x = -2$ and $y = -1$.

109. Solve $x + y = 5$ and $2x - 3y = 4$ by elimination method and the substitution method.

Ans : [Board Term-1 2015]

By Elimination Method :

We have, $x + y = 5$ ...(1)

and $2x - 3y = 4$ ...(2)

Multiplying equation (1) by 3 and adding in (2) we have

$$3(x + y) + (2x - 3y) = 3 \times 5 + 4$$

or, $$3x + 3y + 2x - 3y = 15 + 4$$

$$5x = 19 \Rightarrow x = \frac{19}{5}$$

Substituting $x = \frac{19}{5}$ in equation (1),

$$\frac{19}{5} + y = 5$$

$$y = 5 - \frac{19}{5} = \frac{25 - 19}{5} = \frac{6}{5}$$

Hence, $x = \frac{19}{5}$ and $y = \frac{6}{5}$

By Substituting Method :

We have, $x + y = 5$ ...(1)

and $2x - 3y = 4$ ...(2)

From equation (1), $y = 5 - x$ ...(3)

Substituting the value of y from equation (3) in equation (2),

$$2x - 3(5 - x) = 4$$

$$2x - 15 + 3x = 4$$

$$5x = 19$$

$$x = \frac{19}{5}$$

Substituting this value of x in equation (3), we get

$$y = 5 - \frac{19}{5} = \frac{6}{5}$$

Hence $x = \frac{19}{5}$ and $y = \frac{6}{5}$

110. Solve for x and y :

$$3x + 4y = 10$$

$$2x - 2y = 2$$

Ans : [Board Term-1 2015]

By Elimination Method :

We have, $3x + 4y = 10$ ...(1)

and $2x - 2y = 2$ ...(2)

Multiplying equation (2) by 2 and adding in (1),

$$(3x + 4y) + 2(2x - 2y) = 10 + 2 \times 2$$

or, $$3x + 4y + 4x - 4y = 10 + 4$$

or, $$7x = 14 \Rightarrow x = 2$$

Hence, $x = 2$ and $y = 1$.

By Substitution Method :

We have $3x + 4y = 10$ ...(1)

and $2x - 2y = 2$ ...(2)

From equation (2) $2y = 2x - 2$

or, $y = x - 1$ ...(3)

Substituting this value of y in equation (1),

$$3x + 4(x - 1) = 10$$

$$7x = 14 \Rightarrow x = 2$$

From equation (3), $y = 2 - 1 = 1$

Hence, $x = 2$ and $y = 1$

111. Solve $3x - 5y - 4 = 0$ and $9x = 2y + 7$ by elimination method and the substitution method.

Ans : [Board Term-1 2012]

By Elimination Method :

We have, $3x - 5y = 4$ ...(1)

and $9x = 2y + 7$ ...(2)

Multiplying equation (1) by 3 and rewriting equation (2) we have

$$9x - 15y = 12 \quad ...(3)$$

$$9x - 2y = 7 \quad ...(4)$$

Subtracting equation (4) from equation (3),

$$-13y = 5$$

$$y = -\frac{5}{13}$$

Substituting value of y in equation (1),

$$3x - 5\left(\frac{-5}{13}\right) = 4$$

$$3x = 4 - \frac{25}{13}$$

$$x = \frac{27}{13 \times 3} = \frac{9}{13}$$

Hence $x = \frac{9}{13}$ and $y = -\frac{5}{13}$

By Substituting Method :

We have $3x - 5y = 4$ ...(1)

and $9x = 2y + 7$ ...(2)

$$y = \frac{9x - 7}{2} \quad ...(3)$$

Substituting this value of y (3) in equation (1),

$$3x - 5 \times \left(\frac{9x - 7}{2}\right) = 4$$

$$6x - 45x + 35 = 8$$

$$-39x = -27$$

$$x = \frac{9}{13}$$

Substituting $x = \frac{9}{13}$ in equation (3),

$$y = \frac{9 \times \frac{9}{13} - 7}{2} = \frac{81 - 91}{2 \times 13}$$

$$= -\frac{10}{26} = -\frac{5}{13}$$

Hence, $x = \frac{9}{13}$ and $y = \frac{-5}{13}$

112. A train covered a certain distance at a uniform speed. If the train would have been 10 km/hr scheduled time. And, if the train were slower by 10 km/hr, it would have taken 3 hr more than the scheduled time. Find the distance covered by the train.

Ans : [Board Term-1 2012, NCERT]

Let the actual speed of the train be s and actual time taken t.

$$\text{Distance} = \text{Speed} \times \text{Time}$$

$$= st \text{ km}$$

According to the given condition, we have

$$st = (s + 10)(t - 2)$$

$$st = st - 2s + 10t - 20$$

$$2s - 10t + 20 = 0$$

$$s - 5t = -10 \quad (1)$$

and $st = (s - 10)(t + 3)$

$$st = st + 3s - 10t - 30$$

$$3s - 10t = 30 \quad ...(2)$$

Multiplying equation (1) by 3 and subtracting equation (2) from equation (1),

$$3 \times (s - 5t) - (3s - 10t) = -3 \times 10 - 30$$

$$-5t = -60 \Rightarrow t = 12$$

Substituting value of t equation (1),

$$s - 5 \times 12 = -10$$

$$s = -10 + 60 = 50$$

Hence, the distance covered by the train

$$= 50 \times 12 = 600 \text{ km.}$$

113. The ratio of incomes of two persons is 11:7 and the ratio of their expenditures is 9:5. If each of them manages to save Rs 400 per month, find their monthly incomes.

Ans : [Board Term-1 2012]

Let the incomes of two persons be $11x$ and $7x$.

Also the expenditures of two persons be $9y$ and $5y$.

$$11x - 9y = 400 \quad ...(1)$$

and $7x - 5y = 400$ ...(2)

Multiplying equation (1) by 5 and equation (2) by 9 we have

$$55x - 45y = 2000 \quad ...(3)$$

and $63x - 45y = 3600$ ...(4)

Subtracting, above equation we have

$$-8x = -1600$$

or, $x = \frac{-1,600}{-8} = 200$

Hence Their monthly incomes are $11 \times 200 =$ Rs 2200 and $7 \times 200 =$ Rs 1400.

114. A and B are two points 150 km apart on a highway. Two cars start A and B at the same time. If they move in the same direction they meet in 15 hours. But if they move in the opposite direction, they meet in 1 hours. Find their speeds.

Ans : [Board Term-1 2012]

Let the speed of the car I from A be x and speed of the car II from B be y.

Same Direction :

Distance covered by car I

$= 150 +$(distance covered by car II)

$$15x = 150 + 15y$$

$$15x - 15y = 150$$

$$x - y = 10 \quad ...(1)$$

Opposite Direction :

Distance covered by car I + distance covered by car II

$$= 150 \text{ km}$$

$$x + y = 150 \quad ...(2)$$

Adding equation (1) and (2), we have $x = 80$.

Substituting $x = 80$ in equation (1), we have $y = 70$.

Speed of the car I from $A = 80$ km/hr and speed of the car II from $B = 70$ km/hr.

115. If 2 is subtracted from the numerator and 1 is added to the denominator, a fraction becomes $\frac{1}{2}$, but when 4 is added to the numerator and 3 is subtracted from the denominator, it becomes $\frac{3}{2}$. Find the fraction.

Ans : [Board Term-1 2012]

Let the fraction be $\frac{x}{y}$ then we have

$$\frac{x-2}{y+1} = \frac{1}{2}$$

$$2x - 4 = y + 1$$

$$2x - y = 5 \quad ...(1)1$$

Also, $$\frac{x+4}{y-3} = \frac{3}{2}$$

$$2x + 8 = 3y - 9]$$

$$2x - 3y = -17 \quad ...(2)\ 1$$

Subtracting equation (2) from equation (1),

$$2y = 22 \Rightarrow y = 11$$

Substituting this value of y in equation (1) we have,

$$2x - 11 = 5$$

$$x = 8$$

Hence, Fraction $= \frac{8}{11}$

116. If a bag containing red and white balls, half the number of white balls is equal to one-third the number of red balls. Thrice the total number of balls exceeds seven times the number of white balls by 6. How many balls of each colour does the bag contain ?

Ans : [Board Term-1 2012]

Let the number of red balls be x and white balls be y.
According to the question,

$$\frac{y}{2} = \frac{1}{3}x \text{ or } 2x - 3y = 0 \quad ...(1)$$

and $$3(x + y) - 7y = 6$$

or $$3x - 4y = 6 \quad ...(2)$$

Multiplying equation (1) by 3 and equation (2) by we have

$$6x - 9y = 0 \quad ...(3)$$

$$6x - 8y = 12 \quad ...(4)$$

Subtracting equation (3) from (4) we have

$$y = 12$$

Substituting $y = 12$ in equation (1),

$$2x - 36 = 0$$

$$x = 18$$

Hence, number of red balls $= 18$

and number of white balls $= 12$

117. A two digit number is obtained by either multiplying the sum of digits by 8 and then subtracting 5 or by multiplying the difference of digits by 16 and adding 3. Find the number.

Ans : [Board Term-1 2012]

Let the digits of number be x and y, then number will $10x + y$.
According to the question, we have

$$8(x + y) - 5 = 10x + y$$

$$2x - 7y + 5 = 0 \quad ...(1)$$

also $$16(x - y) + 3 = 10x + y$$

$$6x - 17y + 3 = 0 \quad ...(2)$$

Comparing the equation with $ax + by + c = 0$ we get

$$a_1 = 2, b_1 = -1, c_1 = 5$$

$$a_2 = 6, b_2 = -17, c_2 = 3$$

Now $$\frac{x}{b_2c_1 - b_1c_2} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{c_1b_2 - a_2b_1}$$

$$\frac{x}{(-7)(3) - (-17)(5)} = \frac{y}{(5)(6) - (2)(3)}$$

$$= \frac{1}{(2)(-17) - (6)(-7}$$

$$\frac{x}{-21+85} = \frac{y}{30-6} = \frac{1}{-34+42}$$

$$\frac{x}{64} = \frac{y}{24} = \frac{1}{8}$$

$$\frac{x}{8} = \frac{y}{3} = 1$$

Hence, $x = 8, y = 3$

So required number$= 10 \times 8 + 3 = 83$.

118. The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and the breadth is increased by 3 units. The area is increased by 67 square units if length is increased by 3 units and breadth is increased by 2 units. find the perimeter of the rectangle.

Ans : [Board Term-1 2012, Set-48]

Let length of given rectangle be x and breadth be y, then area of rectangle will be xy.

According to the first condition we have

$$(x-5)(y+3) = xy - 9$$

or, $3x - 5y = 6$ ...(1)

According to the second condition, we have

$$(x+3)(y+2) = xy - 67$$

or, $2x + 5y = 61$ ...(2)

Multiplying equation (1) by 3 and equation (2) by 5 and then adding,

$$9x - 15y = 18$$

$$10x + 15y = 305$$

$$x = \frac{323}{19} = 17$$

Substituting this value of x in equation (1),

$$3(17) - 5y = 6$$

$$5y = 51 - 6$$

$$y = 9$$

Hence, perimeter $= 2(x+y) = 2(17+9) = 52$ units.

119. Solve for x and y : $2(3x-y) = 5xy, 2(x+3y) = 5xy$.

Ans : [Board Term-1 2012, Set-25]

We have $2(3x-y) = 5xy$ ...(1)

$2(x+3y) = 5xy$ ...(2)

Divide equation (1) and (2) by xy,

$$\frac{6}{y} - \frac{2}{x} = 5 \quad ...(3)$$

and $\frac{2}{y} + \frac{6}{x} = 5$ ...(4)

Let $\frac{1}{y} = a$ and $\frac{1}{x} = b$, then equations (3) and (4) become

$6a - 2b = 5$ ...(5)

$2a + 6b = 5$ ...(6)

Multiplying equation (5) by 3 and then adding with equation (6),

$$20a = 20$$

$$a = 1$$

Substituting this value of a in equation (5),

$$b = \frac{1}{2}$$

Now $\frac{1}{y} = a = 1 \Rightarrow y = 1$

and $\frac{1}{x} = b = \frac{1}{2} \Rightarrow x = 2$

Hence, $x = 2, y = 1$

120. The students of a class are made to stand in rows. If 3 students are extra in a row, there would be 1 row less. If 3 students are less in a row, there would be 2 rows more. Find the number of students in the class.

Ans : [Board Term-1 2012, Set-68, NCERT]

Let the number of students in a row be x and the number of rows be y. Thus total will be xy.

Now $(x+3)(y-1) = xy$

$$xy + 3y - x - 3 = xy$$

$-x + 3y - 3 = 0$ ...(1)

and $(x-3)(y+2) = xy$

$$xy - 3y + 2x - 6 = xy$$

$2x - 3y - 6 = 0$ ...(2)

Multiply equation (1) 2 we have

$-2x + 6y - 6 = 0$ ...(3)

Adding equation (2) and (3) we have

$$3y - 12 = 0$$

$$y = 4$$

Substitute $y = 4$ in equation (1)

$$-x + 12 - 3 = 0$$

$$x = 9$$

Total students $xy = 9 \times 4 = 36$

Total students in the class is 36.

121. The ages of two friends ani and Biju differ by 3 years. Ani's father Dharam is twice as old as ani and Biju is twice as old as his sister Cathy. The ages of Cathy and Dharam differ by 30 year. Find the ages of Ani and Biju.

Ans : [Board Term-1 2012, Set-64]

Let the ages of Ani and Biju be x and y, respectively. According to the given condition,

$$x - y = \pm 3 \quad ...(1)$$

Also, age of Ani's father Dharam $= 2x$ years

And age of Biju's sister $= \frac{y}{2}$ years

According to the given condition,

$$2x - \frac{y}{2} = 30$$

$$4x - y = 60 \quad ...(2)$$

Case I : When $x - y = 3$ ...(3)

Subtracting equation (3) from equation (2),

$$3x = 57$$

$$x = 19 \text{ years}$$

Putting $x = 19$ in equation (3),

$$19 - y = 3$$

$$y = 16 \text{ years}$$

Case II : When $x - y = -3$ ...(4)

Subtracting equation (iv) from equation (2),

$$3x = 60 + 3$$

$$3x = 63$$

$$x = 21 \text{ years}$$

Subtracting equation (4), we get

$$21 - y = -3$$

$$y = 24 \text{ years}$$

Hence, Ani's age = 19 years or 21 years Biju age = 16 years or 24 years.

122. One says, "Give me a hundred, friend! I shall then become twice as rich as you." The other replies, "If you give me ten, I shall be six times as rich as you." Tell me what is the amount of their (respective) capital.

Ans : [Board Term-1 2012, Set-54]

Let the amount of their respective capitals be x and y.

According to the given condition,

$$x + 100 = 2(y - 100)$$

$$x - 2y = -300 \quad ...(1)$$

and

$$6(x - 10) = y + 10$$

$$6x - y = 70 \quad ...(2)$$

Multiplying equation (2) by 2 we have

$$12x - 2y = 140 \quad ...(3)$$

Subtracting (1) from equation (3) we have

$$11x = 440$$

$$x = 40$$

Substituting $x = 40$ in equation (1),

$$40 - 2y = -300$$

or,

$$2y = 340$$

$$y = 170$$

Hence, the amount of their respective capitals are 40 and 170.

123. A fraction become $\frac{9}{11}$ if 2 is added to both numerator and denominator. If 3 is added to both numerator and denominator it becomes $\frac{5}{6}$. Find the fraction.

Ans : [Board Term-1 2012, Set-60]

Let the fraction be $\frac{x}{y}$, then according to the question,

$$\frac{x+2}{y+2} = \frac{9}{11}$$

$$11x + 22 = 9y + 18$$

or,

$$11x - 9y + 4 = 0 \quad ...(1)$$

and

$$\frac{x+3}{y+3} = \frac{5}{6}$$

or, $\quad 6x - 5y + 3 = 0 \quad$ ...(2)

Comparing with $ax + by + c = 0$

we get $\quad a_1 = 11, b_1 = 9, c_1 = 4,$

$$a_2 = 6, b_2 = -5, \text{ and } c_2 = 3$$

Now, $$\frac{x}{b_2 c_1 - b_1 c_2} = \frac{y}{c_1 a_2 - c_2 a_1} = \frac{1}{a_1 b_2 - b_2 b_1}$$

$$\frac{x}{(-9)(3) - (-5)(4)} = \frac{y}{(4)(6) - (11)(3)}$$

$$= \frac{1}{(11)(-5) - (9)(-9)}$$

or, $$\frac{x}{-27 + 20} = \frac{y}{24 - 33} = \frac{1}{-55 + 54}$$

$$\frac{x}{-7} = \frac{y}{-9} = \frac{1}{-1}$$

Hence, $x = 7, y = 9$

Thus fraction is $\frac{7}{9}$.

124. A motor boat can travel 30 km upstream and 28 km downstream in 7 hours. It can travel 21 km upstream and return in 5 hours. Find the speed of the boat in still water and the speed of the stream.

Ans : [Board Term-1 2012]

Let the speed of the boat in still water be x km/hr and speed of the stream be y km/hr.

Speed of boat up stream $= (x - y)$ km/hr.

Speed of boat down stream $= (x + y)$ km/hr.

$$\frac{30}{x - y} + \frac{28}{x + y} = 7$$

and $$\frac{21}{x - y} + \frac{21}{x + y} = 5$$

Let $\frac{1}{x - y}$ be a and $\frac{1}{x + y}$ be b, then we have

$$30a + 28b = 7 \quad ...(1)$$

$$21a + 21b = 5 \quad ...(2)$$

Multiplying equation (1) by 3 and equation (2) by 4 we have

$$90a + 84b = 21 \quad ...(3)$$

$$84a + 84b = 20 \quad ...(4)$$

Subtracting (4) from (3) we have,

$$6a = 1$$

$$a = \frac{1}{6}$$

Putting this value of a in equation (1),

$$30 \times \frac{1}{6} + 28b = 7$$

$$28b = 7 - 30 \times \frac{1}{6} = 2$$

$$b = \frac{1}{14}$$

Thus $\quad x + y = 14 \quad$ ...(5)

Now, $$a = \frac{1}{x - y} = \frac{1}{6}$$

or, $\quad x - y = 6 \quad$ ...(6)

and $\quad x + y = 14$

Solving equation (5) and (6), we get

$$x = 10, y = 4$$

Hence, speed of the boat in still water = 10km/hr

and speed of the stream = 4 km/hr.

125. A boat covers 32 km upstream and 36 km downstream in 7 hours. Also, it covers 40 km upstream and 48 km downstream in 9 hours. Find the speed of the boat in still water and that of the stream.

Ans : [Board Term-1 2012, Set-48]

Let the speed of the boat be x km/hr and the speed of the stream be y km/hr.

According to the question,

$$\frac{32}{x - y} + \frac{36}{x + y} = 7$$

and $$\frac{40}{x - y} + \frac{48}{x + y} = 9$$

Let $\frac{1}{x - y} = A, \frac{1}{x + y} = B$, then we have

$$32A + 36B = 7 \quad ...(1)$$

and $\quad 40A + 48B = 9 \quad$ ...(2)

Multiplying equation (1) by 5 and (2) by 4, we have

$$160A + 180B = 35 \quad ...(3)$$

and $\quad 160A + 192B = 36 \quad$ ...(4

Subtracting (4) from (3) we have

$$-12B = -1$$

$$B = \frac{1}{12}$$

Substituting the value of B in (2) we get

$$40A + 48\left(\frac{1}{12}\right) = 9$$

$$40A+4 = 9$$
$$40A = 5$$
$$A = \frac{1}{8}$$

Thus $A = \frac{1}{8}$ and $B = \frac{1}{12}$

Hence $A = \frac{1}{8} = \frac{1}{x-y}$

$$x-y = 8 \quad ...(5)$$

and $B = \frac{1}{12} = \frac{1}{x+y}$

$$x+y = 12 \quad ...(6)$$

Adding equations (5) and (6) we have,

$$2x = 20$$
$$x = 10$$

Substituting this value of x in equation (1),

$$y = x-8 = 10-8 = 2$$

Hence, the speed of the boat in still water = 10 km/hr and speed of the stream = 2 km/hr.

126. For what values of a and b does the following pair of linear equations have infinite number of solution ?

$2x+3y = 7, a(x+y) - b(x-y) = 3a+b-2$

Ans : [Board Term-1 2015]

We have $2x+3y-7 = 0$

Here $a_1 = 2, b_1 = 3, c_1 = -7$

and $a(x+y) - b(x-y) = 3a+b-2$

$$ax+ay-bx+by = 3a+b-2$$
$$(a-b)x+(a+b)y-(3a+b-2) = 0$$

Here $a_2 = a-b, b_2 = a+b, c_2 = -(3a+b-2)$

For infinite many solutions

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$
$$\frac{2}{a-b} = \frac{3}{a+b} = \frac{-7}{(3a+b-2}$$

From $\frac{2}{a-b} = \frac{7}{3a+b-2}$ we have

$$2(3a+b-2) = 7(a-b)$$
$$6a+2b-4 = 7a-7b$$
$$a-9b = -4 \quad ...(1)$$

From $\frac{3}{a+b} = \frac{7}{3a+b-2}$ we have

$$3(3a+b-2) = 7(a+b)$$
$$9a+3b-6 = 7a+7b$$
$$2a-4b = 6$$
$$a-2b = 3 \quad ...(2)$$

Subtracting equation (1) from (2),

$$-7b = -7$$
$$b = 1$$

Substituting the value of b in equation (1),

$$a = 5$$

Hence, $a = 5, b = 1$.

127. At a certain time in a deer, the number of heads and the number of legs of deer and human visitors were counted and it was found that there were 39 heads and 132 legs.

Find the number of deer and human visitors in the park.

Ans : [Board Term-1 2015]

Let the no. of deer be x and no. of human be y. According to the question,

$$x+y = 39 \quad ...(1)$$

and $4x+2y = 132 \quad ...(2)$

Multiply equation (1) from by 2,

$$2x+2y = 78 \quad ...(3)$$

Subtract equation (3) from (2),

$$2x = 54$$
$$x = 27$$

Substituting this value of x in equation (1)

$$27+y = 39$$
$$y = 12$$

So, No. of deer = 27 and No. of human = 12

128. Find the value of p and q for which the system of equations represent coincident lines $2x+3y = 7$,

$(p+q+1)x+(p+2q+2)y=4(p+q)+1$

Ans : [Board Term-1 2012, Set-42]

We have $2x+3y = 7$

$$(p+q+1)x+(p+2q+2)y = 4(p+q)+1$$

Comparing given equation to $ab+by+c=0$ we have
$a_1 = 2, b_1 = 3, c_1 = -7$
$a_2 = p+q+1, b_2 = p+2q+2, c_2 = -4(p+q)-1$
For coincident lines,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{2}{p+q+1} = \frac{3}{p+2q+2} = \frac{7}{4(p+q)+1}$$

From $\frac{3}{p+2q+2} = \frac{7}{4(p+q)+1}$ we have

$$7p+14q+14 = 12p+12q+3$$

$$5p-2q-11 = 0 \quad ...(1)$$

From $\frac{2}{p+q+1} = \frac{7}{4(p+q)+1}$ we have

$$8(p+q)+2 = 7p+7q+7$$

$$8p+8q+2 = 7p+7q+7$$

$$p+q-5 = 0 \quad ...(2)$$

Multiplying equation (2) by 5 we have

$$5p+5q-25 = 0 \quad ...(3)$$

Subtracting equation (1) from (3) we get

$$7q = 14$$

$$q = 2$$

Hence, $p = 3$ and $q=2$.

129. The length of the sides of a triangle are $2x+\frac{y}{2}$, $\frac{5x}{3}+y+\frac{1}{2}$ and $\frac{2}{3}x+2y+\frac{5}{2}$. If the triangle is equilateral , find its perimeter.

Ans : [Board Term-1 2012]

For an equilateral Δ,

$$2x+\frac{y}{2} = \frac{5x}{3}+y+\frac{1}{2} = \frac{1}{2}x+2y+\frac{5}{2}$$

Now $$\frac{4x+y}{2} = \frac{10x+6y+3}{6}$$

$$12x+3y = 10x+6y+3$$

$$2x-3y = 3 \quad ...(1)$$

Again, $$2x+\frac{y}{2} = \frac{2}{3}x+2y+\frac{5}{2}$$

$$\frac{4x+y}{2} = \frac{4x+12y+15}{6}$$

$$12x+3y = 4x+12y+15$$

$$8x-9y = 15 \quad ...(2)$$

Multiplying equation (1) by 3 we have

$$6x-9y = 9 \quad ...(1)$$

Subtracting it from (2) we get

$$2x = 6 \Rightarrow x = 3$$

Substituting this value of x into (1), we get

$$2\times 3-3y = 3$$

or, $$3y = 3 \Rightarrow y = 1$$

Now substituting these value of x and y

$$2x+\frac{y}{2} = 2\times 3+\frac{1}{2} = 6.5$$

The perimeter of equilateral triangle = side $\times 3$

$$= 6.5\times 3 = 19.5 \text{ cm}$$

Hence, the perimeter of $\Delta = 19.5$ m

130. When 6 boys were admitted and 6 girls left, the percentage of boys increased from 60% to 75%. Find the original no. of boys and girls in the class.

Ans : [Board Term-1 2015]

Let the no. of boys be x and no. of girls be y.
No. of students $= x+y$

Now $$\frac{x}{x+y} = \frac{60}{100} \quad ...(1)$$

and $$\frac{x+6}{(x+6)+(y-6)} = \frac{75}{100} \quad ...(2)$$

From (1), we have

$$100x = 60x+60y$$

$$40x-60y = 0$$

$$2x-3y = 0$$

$$2x = 3y \quad (3)$$

From (2) we have

$$100x+600 = 75x+75y$$

$$25x-75y = -600$$

$$x - 3y = -24 \quad ...(4)$$

Substituting the value of $3y$ from (3) in to (4) we have,

$$x - 2x = -24 \Rightarrow x = 24$$

$$3y = 24 \times 2$$

$$y = 16$$

Hence, no. of boys is 24 and no. of girls is 16.

131. A cyclist, after riding a certain distance, stopped for half an hour to repair his bicycle, after which he completes the whole journey of 30 km at half speed in 5 hours. If the breakdown had occurred 10 km farther off, he would have done the whole journey in 4 hours. Find where the breakdown occurred and his original speed.

Ans : [Board Term-1 2013, Set-32]

Let x be the distance of the place where breakdown occurred and y be the original speed,

$$\frac{x}{y} + \frac{30 - x}{\frac{y}{2}} = 5$$

or $$\frac{x}{y} + \frac{60 - 2x}{y} = 5$$

$$x + 60 - 2x = 5y$$

$$x + 5y = 60 \quad ...(1)$$

and $$\frac{x + 10}{y} + \frac{30 - (x + 10)}{\frac{y}{2}} = 4$$

$$\frac{x + 10}{y} + \frac{60 - 2(x + 10)}{y} = 4$$

$$x + 10 + 60 - 2x - 20 = 4y$$

$$-x + 50 = 4y$$

$$x + 4y = 50 \quad (2)$$

Subtract equation (2) from (1), $y = 10$ km/hr.

Now from (2), $$x + 40 = 50$$

$$x = 10 \text{ km}$$

Break down occurred at 10 km and original speed was 10 km/hr.

132. The population of a village is 5000. If in a year, the number of males were to increase by 5% and that of a female by 3% annually, the population would grow to 5202 at the end of the year. Find the number of males and females in the village.

Ans : [Board Term-1 2012, Set-60]

Let the number of males be x and females be y

Now $$x + y = 5{,}000 \quad ...(1)$$

and $$x + \frac{5}{100}x + y + \frac{3y}{100} = 5202$$

$$\frac{5x + 3y}{100} + 5000 = 5202$$

$$5x + 3y = (5202 - 5000) \times 100$$

$$5x + 3y = 20200 \quad (2)$$

Multiply (1) by 3 we have

$$3x + 3y = 15{,}000 \quad ...(3)$$

Subtracting (2) from (3) we have

$$2x = 5200 \Rightarrow x = 2600$$

Substituting value of x in (1) we have

$$2600 - y = 5000$$

$$y = 2400$$

Thus no. of males is 2600 and no. of females is 2400.