CHAPTER 10

CIRCLE

ONE MARK QUESTIONS

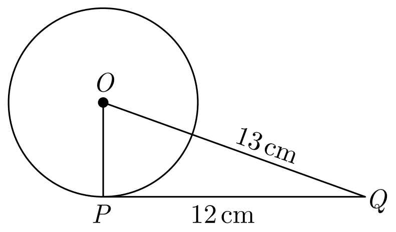
MULTIPLE CHOICE QUESTIONS

1. From an external point Q, the length of tangent to a circle is 12 cm and the distance of Q from the centre of circle is 13 cm. The radius of circle (in cm) is

(a) 10 (b) 5
(c) 12 (d) 7

Ans : [Board 2020 Delhi Basic]

Let O be the centre of the circle. As per given information we have drawn the figure below.



We have $OQ = 13$ cm

and $PQ = 12$ cm

Radius is perpendicular to the tangent at the point of contact.

Thus $OP \perp PQ$

In $\Delta\, OPQ$, using Pythagoras theorem,

$$OP^2 + PQ^2 = OQ^2$$
$$OP^2 + 12^2 = 13^2$$
$$OP^2 = 13^2 - 12^2$$
$$= 169 - 144$$
$$= 25$$

Thus $OP = 5$ cm

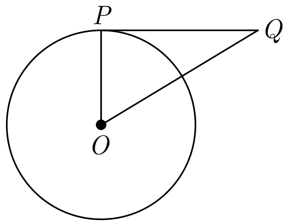
Thus (b) is correct option.

2. QP is a tangent to a circle with centre O at a point P on the circle. If $\Delta\, OPQ$ is isosceles, then $\angle OQR$ equals.

(a) 30° (b) 45°
(c) 60° (d) 90°

Ans : [Board 2020 Delhi Basic]

Let O be the centre of the circle. As per given information we have drawn the figure below.



We know that, the radius and tangent are perpendicular at their point of contact.

Now, in isosceles triangle POQ we have

$$\angle POQ + \angle OPQ + \angle OQP = 180^\circ$$

Equal sides subtend equal angles in isosceles triangle.

Thus $$2\angle OQP + 90^\circ = 180^\circ$$
$$\angle OQP = 45^\circ$$

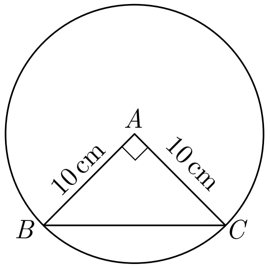
Thus (b) is correct option.

3. A chord of a circle of radius 10 cm, subtends a right angle at its centre. The length of the chord (in cm) is

(a) $\dfrac{5}{\sqrt{2}}$ (b) $5\sqrt{2}$
(c) $10\sqrt{2}$ (d) $10\sqrt{3}$

Ans : [Board 2020 OD Basic]

As per given information we have drawn the figure below.

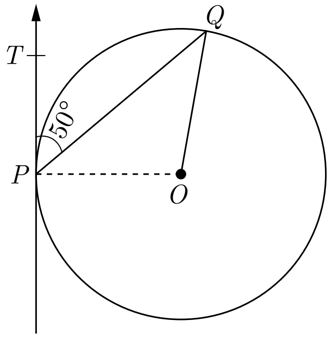


Using Pythagoras theorem in $\Delta\, ABC$, we get

$$BC^2 = AB^2 + AC^2$$
$$= 10^2 + 10^2$$
$$= 100 + 100 = 200$$
$$BC = 10\sqrt{2} \text{ cm}$$

Thus (c) is correct option.

4. In figure, O is the centre of circle. PQ is a chord and PT is tangent at P which makes an angle of 50° with $PQ \angle POQ$ is



(a) 130° (b) 90°
(c) 100° (d) 75°

Ans : [Board 2020 OD Basic]

Due to angle between radius and tangent,

$$\angle OPT = 90^\circ$$
$$\angle OPQ = 90^\circ - 50^\circ = 40^\circ$$

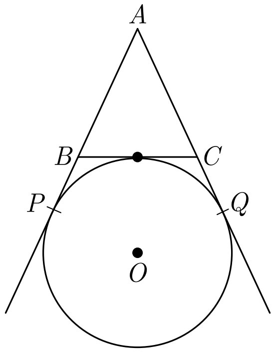
Also, $OP = OQ$ [Radii of a circle]

Since equal opposite sides have equal opposite angles,

$$\angle OPQ = \angle OQP = 40^\circ$$
$$\angle POQ = 180^\circ - \angle OPQ - \angle OQP$$
$$= 180^\circ - 40^\circ - 40^\circ = 100^\circ$$

Thus (c) is correct option.

5. In figure, AP, AQ and BC are tangents of the circle with centre O. If $AB = 5$ cm, $AC = 6$ cm and $BC = 4$ cm, then the length of AP (in cm) is



(a) 15 (b) 10
(c) 9 (d) 7.5

Ans : [Board 2020 Delhi Basic]

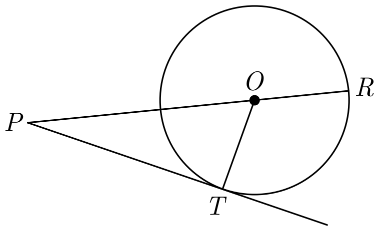
Due to tangents from external points,
$BP = BR, CR = CQ$, and $AP = AQ$
Perimeter of ΔABC,

$$AB + BC + AC$$
$$= AB + BR + RC + AC$$
$$5 + 4 + 6 = AB + BP + CQ + AC$$
$$15 = AP + AQ$$
$$15 = 2AP$$

Thus $$AP = \frac{15}{2} = 7.5 \text{ cm}$$

Thus (d) is correct option.

6. In figure, on a circle of radius 7 cm, tangent PT is drawn from a point P such that $PT = 24$ cm. If O is the centre of the circle, then the length of PR is



(a) 30 cm (b) 28 cm
(c) 32 cm (d) 25 cm

Ans : [Board 2020 Delhi Basic]

Tangent at any point of a circle is perpendicular to the radius at the point of contact.

Thus $$OT \perp PT$$

Now in right-angled triangle PTO

$$OP^2 = OT^2 + PT^2$$

$$= (7)^2 + (24)^2$$

$$= 49 + 576$$

$$= 625$$

Thus $OP = 25$ cm

Since $OR = OT$ because of radii of circle,

$$PR = OP + OR = 25 + 7 = 32 \text{ cm}$$

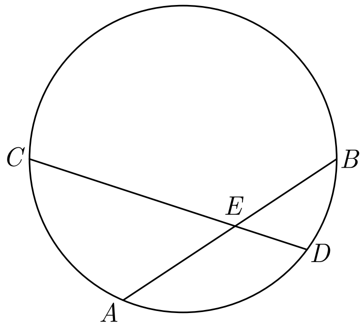
Thus (c) is correct option.

7. Two chords AB and CD of a circle intersect at E such that $AE = 2.4$ cm, $BE = 3.2$ cm and $CE = 1.6$ cm. The length of DE is

(a) 1.6 cm (b) 3.2 cm

(c) 4.8 cm (d) 6.4 cm

Ans : (c) 4.8 cm



Applying the rule, $AE \times EB = CE \times ED$

$$2.4 \times 3.2 = 1.6 \times ED$$

$$ED = 4.8 \text{ cm}$$

Thus (c) is correct option.

8. If a regular hexagon is inscribed in a circle of radius r, then its perimeter is

(a) $3r$ (b) $6r$

(c) $9r$ (d) $12r$

Ans :

Side of the regular hexagon inscribed in a circle of radius r is also r, the perimeter is $6r$.

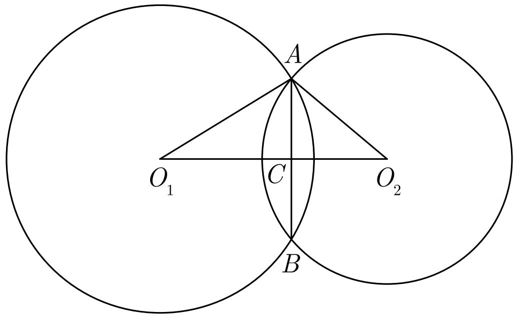
Thus (b) is correct option.

9. Two circles of radii 20 cm and 37 cm intersect in A and B. If O_1 and O_2 are their centres and $AB = 24$ cm, then the distance O_1O_2 is equal to

(a) 44 cm (b) 51 cm

(c) 40.5 cm (d) 45 cm

Ans :



Since C is the mid-point of AB,

$$AC = 12$$

$$AO_1 = 37$$

and $AO_2 = 20$

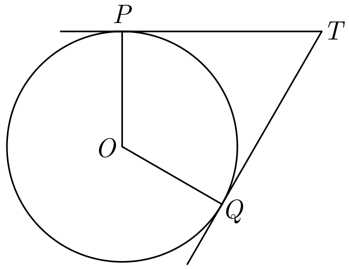
$$CO_1 = \sqrt{37^2 - 12^2} = 35$$

$$CO_2 = \sqrt{20^2 - 12^2} = 16$$

$$O_1O_2 = 35 + 16 = 51$$

Thus (b) is correct option.

10. In the adjoining figure, TP and TQ are the two tangents to a circle with centre O. If $\angle POQ = 110^\circ$, then $\angle PTQ$ is



(a) 60° (b) 70°

(c) 80° (d) 90°

Ans :

Here $OP \perp PT$ and $OQ \perp QT$,

In quadrilateral $OPTQ$, we have

$$\angle POQ + \angle OPT + \angle PTQ + \angle OQT = 360^\circ$$

$$110^\circ + 90^\circ + \angle PTQ + 90^\circ = 360^\circ$$

$$\angle PTQ = 70^\circ$$

Thus (b) is correct option.

11. AB and CD are two common tangents to circles which touch each other at a point C. If D lies on AB such that $CD = 4\,\text{cm}$ then AB is

(a) 12 cm (b) 8 cm

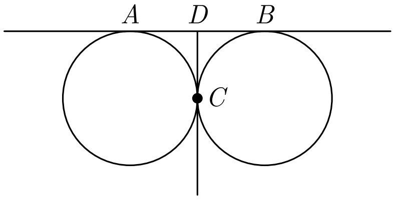
(c) 4 cm (d) 6 cm

Ans :

$$AD = CD \text{ and } BD = CD$$

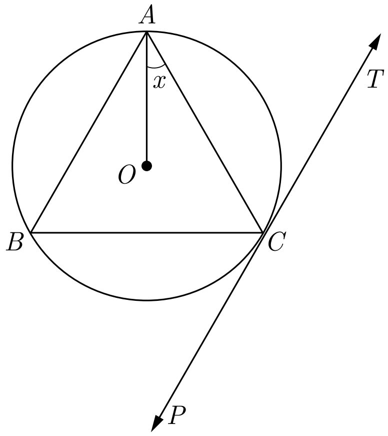
$$AB = AD + BD = CD + CD$$

$$= 2CD = 2 \times 4 = 8\,\text{cm}$$



Thus (b) is correct option.

12. In the adjoining figure, PT is a tangent at point C of the circle. O is the circumference of ΔABC. If $\angle ACP = 118°$, then the measure of $\angle x$ is

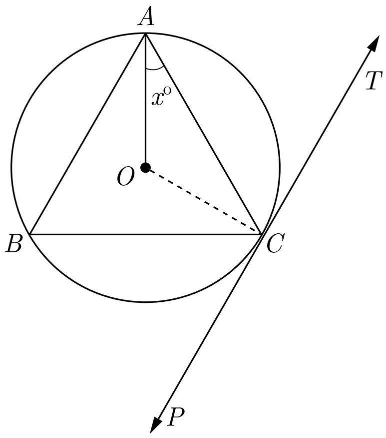


(a) $28°$ (b) $32°$

(c) $42°$ (d) $38°$

Ans :

We join OC as shown in the below figure. Here OC is the radius and PT is the tangent to circle at point C.



Thus $OC \perp PT$

$$\angle OCP = 90°$$

Given, $\angle ACP = 118°$

$$\angle ACO = \angle ACP - \angle OCP$$

$$= 118° - 90° = 28°$$

$$\angle ACO = 28°$$

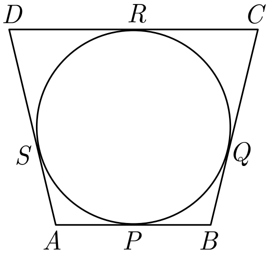
Since O is the circumcentre, thus $OA = OC$ (radius)

$$\angle OAC = \angle ACO$$

$$x = 28°$$

Thus (a) is correct option.

13. In the given figure, a circle touches all the four sides of quadrilateral $ABCD$ with $AB = 6\,\text{cm}$, $BC = 7\,\text{cm}$ and $CD = 4\,\text{cm}$, then length of AD is



(a) 3 cm (b) 4 cm

(c) 5 cm (d) 6 cm

Ans :

Four sides of a quadrilateral $ABCD$ are tangent to a circle.

$$AB + CD = BC + AD$$

$$6 + 4 = 7 + AD$$

$$AD = 10 - 7 = 3 \text{ cm}$$

Thus (a) is correct option.

14. Two concentric circles of radii a and b where $a > b$, The length of a chord of the larger circle which touches the other circle is

(a) $\sqrt{a^2 + b^2}$ (b) $2\sqrt{a^2 + b^2}$

(c) $\sqrt{a^2 - b^2}$ (d) $2\sqrt{a^2 - b^2}$

Ans :

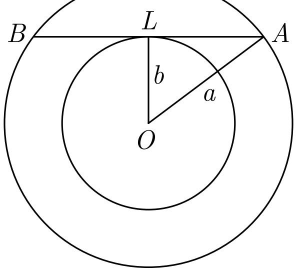
In $\Delta\, OAL$, $$OA^2 = OL^2 + AL^2$$

$$a^2 = OL^2 + b^2$$

$$OL = \sqrt{a^2 - b^2}$$

Length of chord,

$$2AL = 2\sqrt{a^2 - b^2}$$



Thus (d) is correct option.

15. Two concentric circles are of radii 10 cm and 8 cm, then the length of the chord of the larger circle which touches the smaller circle is

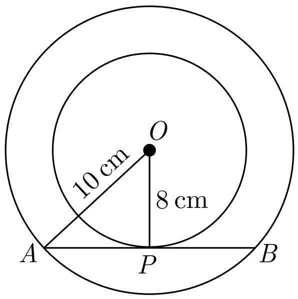
(a) 6 cm (b) 12 cm

(c) 18 cm (d) 9 cm

Ans :

Let O be the centre of the concentric circles of radii 10 cm and 8 cm, respectively. Let AB be a chord of the larger circle touching the smaller circles at P.

Then, $$AP = PB \text{ and } OP \perp AB$$



Applying Pythagoras theorem in $\Delta\, OPA$, we have

$$OA^2 = OP^2 + AP^2$$

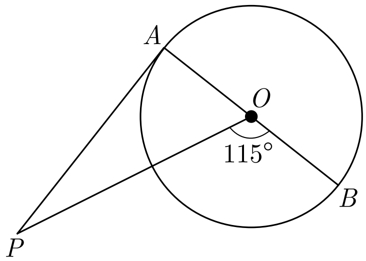
$$100 = 64 + AP^2$$

$$AP^2 = 100 - 64 = 36 \Rightarrow AP = 6 \text{ cm}$$

$$AB = 2AP = 2 \times 6 = 12 \text{ cm}$$

Thus (b) is correct option.

16. In the given figure, PA is a tangent from an external point P to a circle with centre O. If $\angle POB = 115^\circ$, then perimeter of $\angle APO$ is



(a) 25° (b) 20°

(c) 30° (d) 65°

Ans :

Since tangent at a point to a circle is perpendicular to the radius,

$$\angle OAP = 90^\circ$$

$$\text{Now,} \quad \angle AOP + \angle BOP = 180^\circ$$
$$\angle AOP + 115^\circ = 180^\circ$$
$$\angle AOP = (180^\circ - 115^\circ) = 65^\circ$$

From angle sum property of triangle we have

$$\angle OAP + \angle AOP + \angle APO = 180^\circ$$
$$90^\circ + 65^\circ + \angle APO = 180^\circ$$
$$155^\circ + \angle APO = 180^\circ$$
$$\angle APO = 180^\circ - 155^\circ = 25^\circ$$

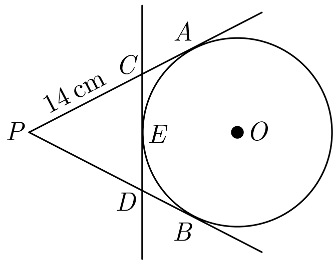
Thus (a) is correct option.

17. From an external point P, tangents PA and PB are drawn to a circle with centre O. If CD is the tangent to the circle at a point E and $PA = 14\,\text{cm}$. The perimeter of $\Delta\,PCD$ is

(a) 14 cm (b) 21 cm
(c) 28 cm (d) 35 cm

Ans :

As per information given in question we have drawn figure below.



$$\text{Here} \quad PA = PB = 14\,\text{cm}$$

Also, CD is tangent at point E on the circle.

So, CA and CE are tangent to the circle from point C.

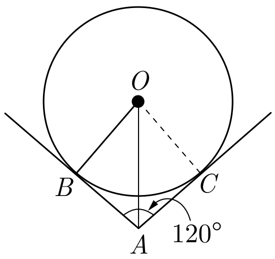
$$\text{Therefore,} \quad CA = CE,$$
$$\text{Similarly,} \quad DB = DE$$

Now, perimeter of $\Delta\,PCD$,

$$PC + CD + PD = PC + CE + ED + PD$$
$$= PC + CA + PD + DB$$
$$= PA + PB$$
$$= 14 + 14$$
$$= 28 \text{ cm}$$

Thus (c) is correct option.

18. In the given figure, two tangents AB and AC are drawn to a circle with centre O such that $\angle BAC = 120^\circ$, then OA is equal to that



(a) $2AB$ (b) $3AB$
(c) $4AB$ (d) $5AB$

Ans :

In $\Delta\,OAB$ and $\Delta\,OAC$, we have,

$$\angle OBA = \angle OCA = 90^\circ$$
$$OA = OA \qquad \text{[common]}$$
$$\text{and} \quad OB = OC \qquad \text{[radii of circle]}$$

So, by RHS congruence criterion,

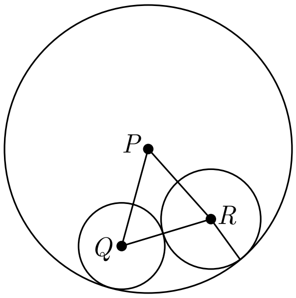
$$\Delta\,OBA \cong \Delta\,OCA$$
$$\angle OAB = \angle OAC = \frac{1}{2} \times 120^\circ = 60^\circ$$

In $\Delta\,OBA$, we have,

$$\cos 60^\circ = \frac{AB}{OA}$$
$$\frac{1}{2} = \frac{AB}{OA}$$
$$OA = 2AB$$

Thus (a) is correct option.

19. In the given figure, three circles with centres P, Q and R are drawn, such that the circles with centres Q and R touch each other externally and they touch the circle with centre P, internally. If $PQ = 10\,\text{cm}$, $PR = 8\,\text{cm}$ and $QR = 12\,\text{cm}$, then the diameter of the largest circle is:



(a) 30 cm (b) 20 cm

(c) 10 cm (d) None of these

Ans :

Let radii of the circles with centres P, Q and R are p, q and r, respectively.

Then, $PQ = p - q = 10$ ...(1)

$PR = p - r = 8$ ...(2)

and $QR = q + r = 12$ ...(3)

Adding equation (2) and (3), we get,

$p + q = 20$ ...(4)

Adding equation (1) and (4), we get,

$2p = 30$

Hence, diameter of the largest circle $2p = 30$.

Thus (a) is correct option.

20. If radii of two concentric circles are 4 cm and 5 cm, then the length of each of one circle which is tangent to the other circle, is

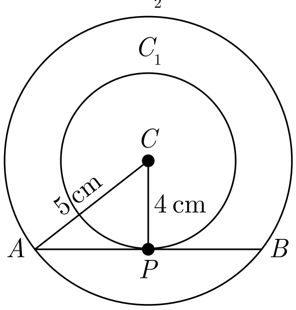
(a) 3 cm (b) 6 cm

(c) 9 cm (d) 1 cm

Ans :

Let C be the centre of two concentric circles C_1 and C_2, whose radii are $r_1 = 4$ cm and $r_2 = 5$ cm.

Now, we draw a chord $AB$of circle C_2, which touches C_1 at P.



AB is tangent at P and CP is radius at P. Tangent at any point of circle is perpendicular to the radius through the point of contact.

Thus $CP \perp AB$

Now, in right triangle PAC

By Pythagoras theorem we have

$$AP^2 = AC^2 - PC^2 = 5^2 - 4^2 = 25 - 16 = 9$$

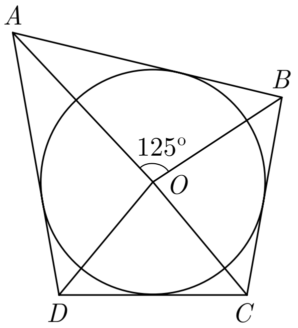
$$AP = 3\text{ cm}$$

So, length of chord,

$$AB = 2AP = 2 \times 3 = 6\text{ cm}$$

Thus (b) is correct option.

21. In figure, if $\angle AOB = 125°$, then $\angle COD$ is equal to



(a) 62.5° (b) 45°

(c) 35° (d) 55°

Ans :

We know that, a quadrilateral circumscribing a circle subtends supplementary angles at the centre of the circle.

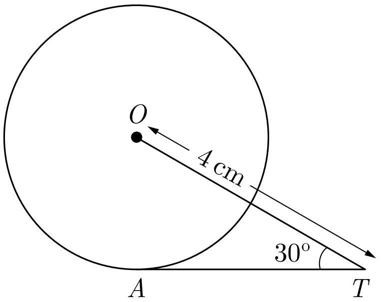
i.e. $\angle AOB + \angle COD = 180°$

$125° + \angle COD = 180°$

$\angle COD = 180° - 125° = 55°$

Thus (d) is correct option.

22. In figure, AT is a tangent to the circle with centre O such that $OT = 4$ cm and $\angle OTA = 30°$. Then, AT is equal to



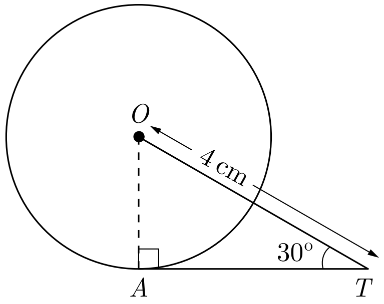
(a) 4 cm (b) 2 cm

(c) $2\sqrt{3}$ cm (d) $4\sqrt{3}$ cm

Ans :

First we joint OA. The tangent at any point of a circle is perpendicular to the radius through the point of contact.

$$\angle OAT = 90^\circ \text{ and } OT = 4 \text{ cm (given)}$$



In ΔOAT, $\cos 30^\circ = \frac{AT}{OT}$

$$\frac{AT}{4} = \frac{\sqrt{3}}{2}$$

$$AT = 2\sqrt{3} \text{ cm}$$

Thus (c) is correct option.

23. Assertion : If in a circle, the radius of the circle is 3 cm and distance of a point from the centre of a circle is 5 cm, then length of the tangent will be 4 cm.

Reason : $(\text{hypotenuse})^2 = (\text{base})^2 + (\text{height})^2$

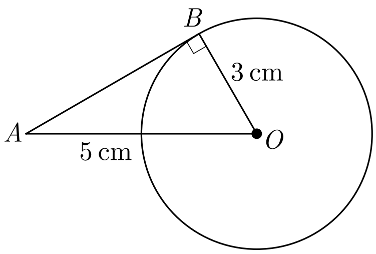
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :



$$OA^2 = AB^2 + OB^2$$

$$5^2 = AB^2 + 3^2$$

$$AB = \sqrt{25-9} = 4 \text{ cm}$$

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Thus (a) is correct option.

24. Assertion : The two tangents are drawn to a circle from an external point, then they subtend equal angles at the centre.

Reason : A parallelogram circumscribing a circle is a rhombus.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

From an external point the two tangents drawn subtend equal angles at the centre.

So assertion is true.

Also, a parallelogram circumscribing a circle is a rhombus, so reason is also true but R is not correct explanation of A.

Thus (b) is correct option.

25. Assertion : PA and PB are two tangents to a circle with centre O. Such that $\angle AOB = 110^\circ$, then $\angle APB = 90^\circ$.

Reason : The length of two tangents drawn from an external point are equal.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

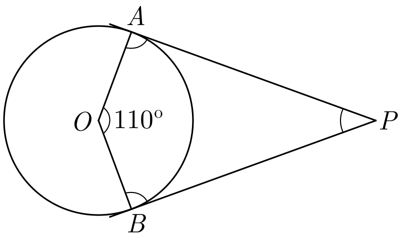
(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

Ans : (d) Assertion (A) is false but reason (R) is true.

As per information given in question we have drawn figure below.



Radius is perpendicular to the tangent at point of contact.

Thus, $OA \perp AP$ and $OB \perp PB$.

In quadrilateral, $OAPB$, we have

$$\angle OAP + \angle APB + \angle PBO + \angle AOB = 360^\circ$$

$$90^\circ + \angle APB + 90^\circ + 110^\circ = 360^\circ$$

$$\angle APB = 70^\circ$$

Assertion (A) is false but reason (R) is true.

Thus (d) is correct option.

Fill in the Blank Questions

26. The lengths of the two tangents from an external point to a circle are ....

Ans :

parallel

27. A line that intersects a circle in one point only is called ....

Ans :

tangent

28. The tangents drawn at the ends of a diameter of a circle are ....

Ans :

two

29. A tangent of a circle touches it at .... point(s).

Ans :

one

30. Tangent is perpendicular to the .... through the point of contact.

Ans :

radius

31. A line intersecting a circle at two points is called a ....

Ans :

secant

32. A circle can have .... parallel tangents at the most.

Ans :

two

33. The common point of a tangent to a circle and the circle is called ....

Ans :

point of contact

34. There is no tangent to a circle passing through a point lying .... the circle.

Ans :

inside

35. The tangent to a circle is .... to the radius through the point of contact.

Ans :

perpendicular

36. There are exactly two tangents to a circle passing through a point lying .... the circle.

Ans :

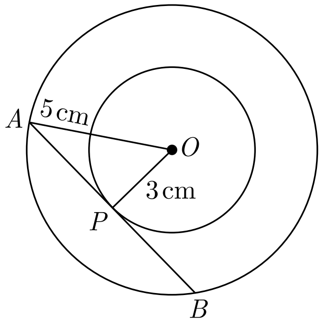
outside equal

37. Length of two tangents drawn from an external point are ....

Ans :

equal

38. In given figure, the length $PB =$ .... cm.



Ans : [Board 2020 OD Standard]

We have $AO = 5\text{ cm}$

and $OP = 3\text{ cm}$

Since AB is a tangent at P and OP is radius, we have

$$\angle APO = 90^\circ$$

In right angled $\Delta\, OPA$,

$$AP^2 = AO^2 - OP^2$$

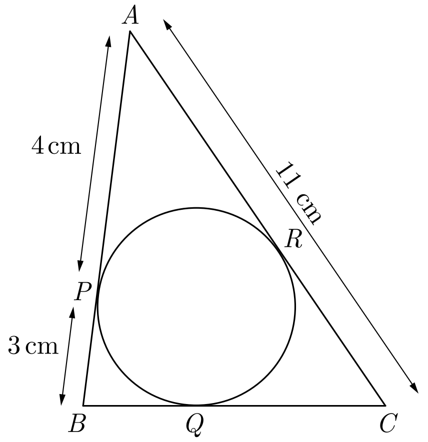
$$= (5)^2 - (3)^2 = 25 - 9 = 16$$

$$AP = 4\,\text{cm}$$

Perpendicular from centre to chord bisect the chord. Thus

$$AP = BP = 4\,\text{cm}$$

39. In figure, $\Delta\, ABC$ is circumscribing a circle, the length of BC is .... cm.



Ans : [Board 2020 Delhi Standard]

Since AP and AR are tangents to the circle from external point A, we have

$$AP = AR = 4\text{ cm}$$

Similarly, PB and BQ are tangents.

Therefore $\quad BP = BQ = 3\,\text{cm}$

Now, $\quad CR = AC - AR = 11 - 4 = 7\,\text{cm}$

Similarly, CR and CQ are tangents.

Therefore $\quad CR = CQ = 7\,\text{cm}$

Now, $\quad BC = BQ + CQ = 3 + 7 = 10\,\text{cm}$

Hence, the length of BC is 10 cm.

Very Short Answer Questions

40. If the angle between two radii of a circle is 130°, then what is the angle between the tangents at the end points of radii at their point of intersection ?

Ans : [Board Term-2 2012]

Sum of the angles between radii and between intersection point of tangent is always 180°.

Thus angle at the point of intersection of tangents

$$= 180^\circ - 130^\circ = 50^\circ$$

41. To draw a pair of tangents to a circle which are inclined to each other at an angle of 30°, it is required to draw tangents at end points of two radii of the circle, what will be the angle between them ?

Ans : [Board Term-2 2012]

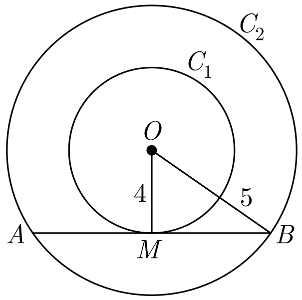
Sum of the angles between radii and between intersection point of tangent is always 180°.

$$\text{Angle between the radii} = 180^\circ - 30^\circ = 150^\circ$$

42. If the radii of two concentric circle are 4 cm and 5 cm, then find the length of each chord of one circle which is tangent to the other circle.

Ans :

As per given information we have drawn the figure below.



Since chord AB is tangent to circle C_1 at point M,

$$OM \perp AB$$

In $\Delta\, OMB$, $\quad OB^2 = OM^2 + MB^2$

$$25 = 4^2 + MB^2$$

$$MB^2 = 25 - 16 = 9$$

$$MB = 3$$

Since, $OM \perp AB$, we obtain $AM = MB$

Now, $\quad AB = 2MB = 2 \times 3 = 6\,\text{cm}$

Hence, length of chord is 6 cm.

43. If a circle can be inscribed in a parallelogram how will the parallelogram change?

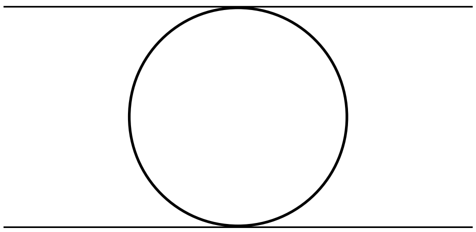
Ans : [Board Term-2, 2014]

It changes into a rectangle or a square.

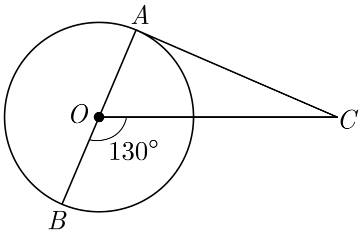
44. What is the maximum number of parallel tangents a circle can have on a diameter?

Ans : [Board Term-2 2012]

Tangent touches a circle on a distinct point. Only two parallel tangents can be drawn on the diameter of a circle. It has been shown in figure given below.



45. In the given figure, AOB is a diameter of the circle with centre O and AC is a tangent to the circle at A. If $\angle BOC = 130°$, the find $\angle ACO$.



Ans : [Board Term-2 Foreign 2016]

Here OA is radius and AC is tangent at A, since radius is always perpendicular to tangent, we have

$$\angle OAC = 90^{\circ}$$

From exterior angle property,

$$\angle BOC = OAC + \angle ACO$$

$$130^{\circ} = 90° + \angle ACO$$

$$\angle ACO = 130^{\circ} - 90^{\circ} = 40^{\circ}$$

46. If a line intersects a circle in two distinct points, what is it called ?

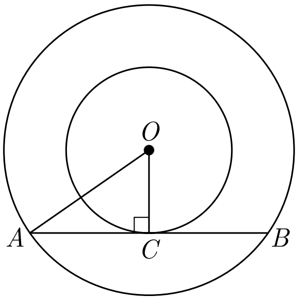
Ans : [Board Term-2, 2012]

The line which intersects a circle in two distinct points is called secant.

47. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of larger circle (in cm) which touches the smaller circle.

Ans : [Board 2020 OD Basic, Foreign 2014]

As per the given question we draw the figure as below.



Here AB is the chord of large circle which touch the smaller circle at point C. We can see easily that ΔAOC is right angled triangle.

Here, $AO = 5$ cm, $OC = 3$ cm

$$AC = \sqrt{AO^2 - OC^2}$$

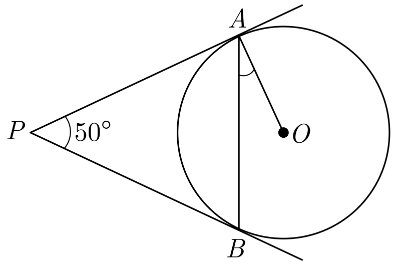
$$= \sqrt{5^2 - 3^2}$$

$$= \sqrt{25 - 9} = \sqrt{16} = 4$$

cm

Length of chord, $AB = 8$ cm.

48. In figure, PA and PB are tangents to the circle with centre O such that $\angle APB = 50°$. Write the measure of $\angle OAB$.



Ans : [[Board Term-2 Delhi 2015]

We have $\angle APB = 50^{\circ}$

$$\angle PAB = \angle PBA = \frac{180^{\circ} - 50^{\circ}}{2} = 65^{\circ}$$

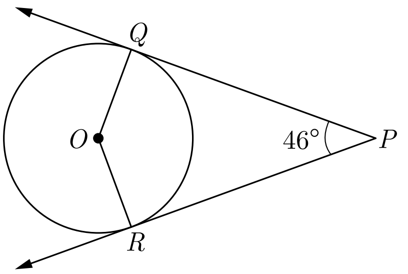
Here OA is radius and AP is tangent at A, since radius is always perpendicular to tangent at point of contact, we have

$$\angle OAP = 90^{\circ}$$

Now $\angle OAB = \angle OAP - \angle PAB$

$$= 90^{\circ} - 65^{\circ} = 25^{\circ}$$

49. If PQ and PR are two tangents to a circle with centre O. If $\angle QPR = 46^{\circ}$ then find $\angle QOR$.



Ans : [Board Term-2 Delhi 2014]

We have $\angle QPR = 46^{\circ}$

Since $\angle QOR$ and $\angle QPR$ are supplementary angles

$$\angle QOR + \angle QPR = 180^{\circ}$$

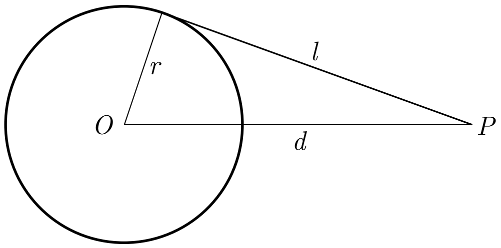
$$\angle QOR + 46^{\circ} = 180^{\circ}$$

$$\angle QOR = 180^{\circ} - 46^{\circ} = 134^{\circ}$$

50. What is the length of the tangent drawn from a point 8 cm away from the centre of a circle of radius 6 cm ?

Ans : [Board Term-2, 2012]

As per the given question we draw the figure as below.



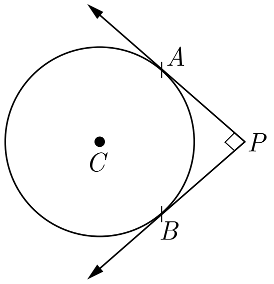
Length of the tangent, $l = \sqrt{d^2 - r^2}$

$$= \sqrt{8^2 - 6^2}$$

$$= \sqrt{64 - 36}$$

$$= \sqrt{28} = 2\sqrt{3} \text{ cm.}$$

51. In figure, PA and PB are two tangents drawn from an external point P to a circle with centre C and radius 4 cm. If $PA \perp PB$, then find the length of each tangent.



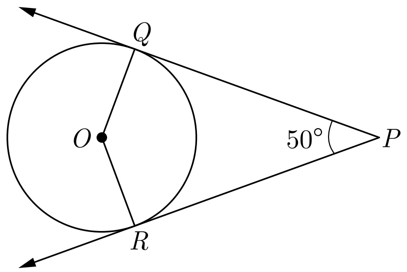
Ans : [Board Term-2, 2013]

Here tangent drawn on circle from external point P are at aright angle, $CAPB$ will be a square.

Thus $CA = AP = PB = BC = 4$ cm

Thus length of tangent is 4 cm.

52. In the given figure, PQ and PR are tangents to the circle with centre O such that $\angle QPR = 50^{\circ}$, Then find $\angle OQR$.



Ans : [Board Term-2 Delhi 2012, 2015]

We have $\angle QPR = \angle 50^{\circ}$ (Given)

Since $\angle QOR$ and $\angle QPR$ are supplementary angles

$$\angle QOR + \angle QPR = 180^{\circ}$$

$$\angle QOR = 180^{\circ} - \angle QPR$$

$$= 180^{\circ} - 50^{\circ} = 130^{\circ}$$

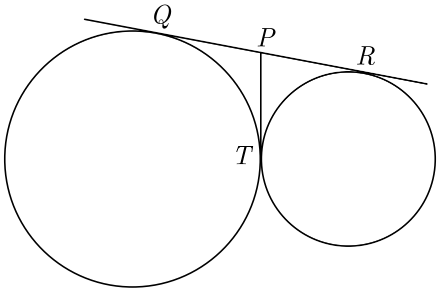
From $\Delta\, OQR$ we have

$$\angle OQR = \angle ORQ = \frac{180^{\circ} - 130^{\circ}}{2}$$

$$= \frac{50^{\circ}}{2} = 25^{\circ}$$

53. In the figure, QR is a common tangent to given circle which meet at T. Tangent at T meets QR at P. If

$QP = 3.8$ cm, then find length of QR.



Ans : [Board Term-2 Delhi 2012, 2014]

Let us first consider large circle. Since length of tangents from external points are equal, we can write

$$QP = PT$$

Thus $$QP = PT = 3.8 \quad ....(1)$$

Now consider the small circle. For this circle we can also write using same logic,

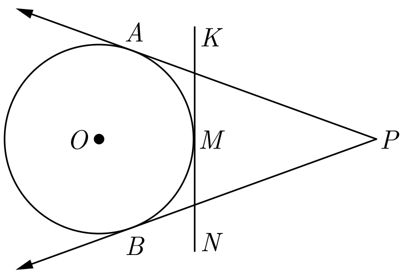
$$PR = PT$$

But we have $PT = 3.8$ cm

Thus $$PR = PT = 3.8 \text{ cm}$$

Now $$QR = QP + PR$$
$$= 3.8 + 3.8 = 7.6 \text{ cm}.$$

54. PA and PB are tangents from point P to the circle with centre O as shown in figure. At point M, a tangent is drawn cutting PA at K and PB at N. Prove that $KN = AK + BN$

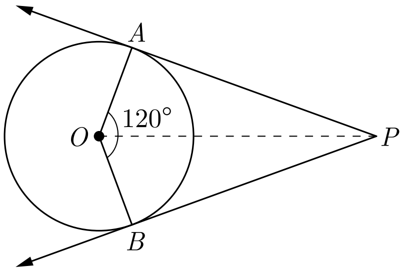


Ans :

Since length of tangents from an external point to a circle are equal,

$$PA = PB, KA = KM, NB = NM,$$
$$KA + NB = KM + NM$$
$$AK + BN = KN. \qquad \text{Hence Proved}$$

55. In the figure, PA and PB are tangents to a circle with centre O. If $\angle AOB = 120°$, then find $\angle OPA$.



Ans : [Board Term-2 Delhi 2012, 2014]

Here OA is radius and AP is tangent at A, since radius is always perpendicular to tangent at point of contact, we have

$$\angle OAP = 90^o$$

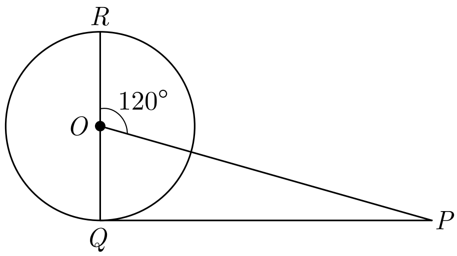
Due to symmetry we have

$$\angle AOP = \frac{\angle AOB}{2} = \frac{120°}{2} = 60°$$

Now in right ΔAOP we have

$$\angle APO + \angle OAP + \angle AOP = 180°$$
$$\angle APO + 90° + 60° = 180°$$
$$\angle APO = 180° - 150° = 30°.$$

56. PQ is a tangent drawn from an external point P to a circle with centre O, QOR is the diameter of the circle. If $\angle POR = 120°$, What is the measure of $\angle OPQ$?



Ans : [Board Term-2 Foreign 2017]

Since PQ is a tangent to the circle, ΔOQP is right angle triangle

In $\Delta\, OQP$ because of exterior angle,

$$\angle POR = \angle OQP + \angle OPQ$$

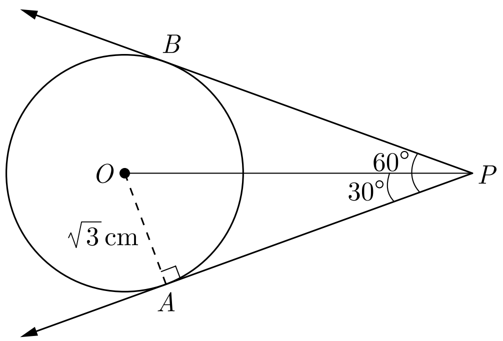
Thus $$\angle OPQ = \angle POR - \angle OQP$$

$$= 120^\circ - 90^\circ = 30^\circ$$

57. Two tangents making an angle of 60° between them are drawn to a circle of radius $\sqrt{3}$ cm, then find the length of each tangent.

Ans : [Board, Term-2, 2013]

As per the given question we draw the figure as below.



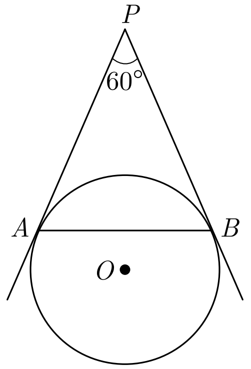
Since, $$\tan\theta = \frac{OA}{AP}$$

So. $$\tan 30^\circ = \frac{OA}{AP}$$

$$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{AP}$$

$$AP = \sqrt{3}\times\sqrt{3} = 3 \text{ cm.}$$

58. In figure, AP and BP are tangents to a circle with centre O, such that $AP = 5$ cm and $\angle APB = 60^\circ$. Find the length of chord AB.



Ans : [Board Term-2 Delhi 2016]

Since length of 2 tangents drawn from an external point to a circle are equal, we have

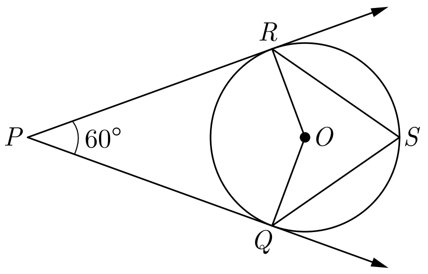
$$PA = PB$$

Thus $$\angle PAB = \angle PBA = 60^\circ$$

Hence $\Delta\, PAB$ is an equilateral triangle.

Therefore $AB = PA = 5$ cm.

59. In the given figure, find $\angle QSR$.



Ans : [Board Term-2, 2012]

Sum of the angles between radii and between intersection point of tangent is always 180°.

Thus $$\angle ROQ + \angle RPQ = 180^\circ$$

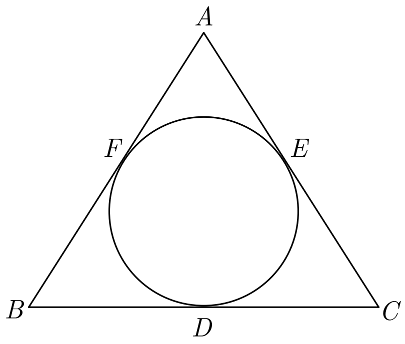
$$\angle ROQ = 180^\circ - 60^\circ = 120^\circ$$

We know that angle subtended on the centre of a circle is twice of the angle subtended on circumference of circle

Thus $$\angle QSR = \frac{1}{2}\angle ROQ = \frac{1}{2}\times 120^\circ$$

$$= 60^\circ$$

60. A triangle ABC is drawn to circumscribe a circle. If $AB = 13$ cm, $BC = 14$ cm and $AE = 7$ cm, then find AC.



Ans : [Board Term-2 Delhi 2012]

Since AF and AE are tangent of the circle, $AF = AE$

Thus $AF = AE = 7$ cm

Now $BF = AB - AF = 13 - 7 = 6$ cm

Since BF and BD are tangent of the circle, $BF = BD$

Thus $BD = BF = 6$ cm

Now $CD = BC - BD = 14 - 6 = 8$ cm

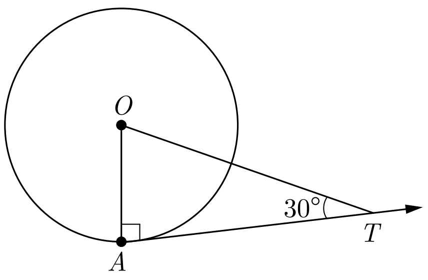
Since CD and CE are tangent of the circle, $CD = CE$

Thus $CE = CD = 8$ cm

Now $AC = AE + EC$

$= 7 + 8 = 15$ cm.

61. In given figure, if AT is a tangent to the circle with centre O, such that $OT = 4$ cm and $\angle OTA = 30°$, then find the length of AT (in cm).



Ans : [Board Term-2, 2012 Set (13)]

Since AT is a tangent to the circle, ΔOAT is right angle triangle

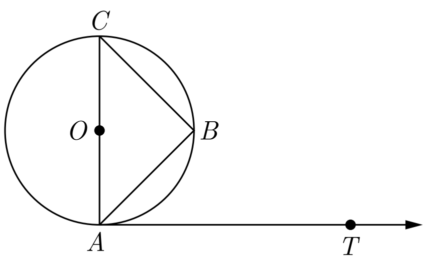
Now $\cos 30° = \frac{AT}{OT}$

$AT = OT \cos 30°$

$= 4 \times \frac{\sqrt{3}}{2} = 2\sqrt{3}$ cm.

Thus the length of AT is $2\sqrt{3}$ cm.

62. In the given figure, AB is a chord of the circle and AOC is its diameter such that $\angle ACB = 50°$. If AT is the tangent to the circle at the point A, find $\angle BAT$.



Ans : [Board Term-2 2012]

We have $\angle ACB = 50°$

Since $\angle CBA$ is angle in semi-circle,

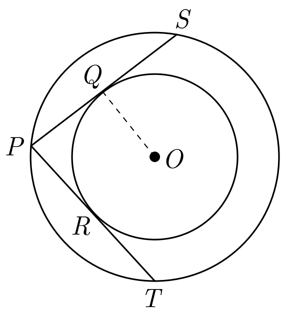
$\angle CBA = 90°$

Now $\angle OAB = 180° - 90° - 50°$
$= 40°$

$\angle BAT = 90° - \angle OAB$

$= 90° - 40° = 50°$

63. In the figure there are two concentric circles with centre O. PRT and PQS are tangents to the inner circle from a point P lying on the outer circle. If $PR = 5$ cm find the length of PS.



Ans : [Board Term-2 Delhi Compt. 2017]

Since PQ and PR are tangent of the circle, $PQ = PR$

$PQ = PR = 5$ cm

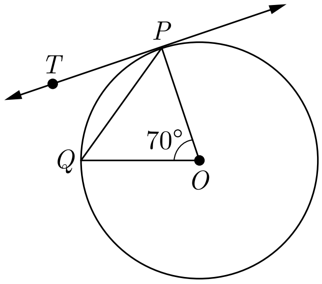
Since PS is chord of circle and point Q bisect it, thus

$PQ = QS$

$PS = 2PQ$

$= 2 \times 5 = 10$ cm

64. In figure, O is the centre of the circle, PQ is a chord and PT is tangent to the circle at P.



Ans : [Board Term-2 OD 2017]

We have $\angle OPQ = \angle OQP$

$$= \frac{180^\circ - 70^\circ}{2} = 55^\circ$$

Thus $TPQ = 90^\circ - 55^\circ = 35^\circ$

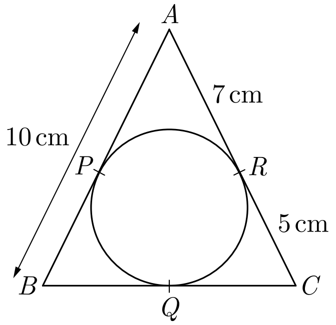
TWO MARKS QUESTIONS

65. A circle is inscribed in a ΔABC touching AB, BC and AC at P, Q and R respectively. If $AB = 10$ cm $AR = 7$ cm and $CR = 5$ cm, then find the length of BC

Ans : [Board 2020 OD Basic]

As per given information we have drawn the figure below.

Here a circle is inscribed in a ΔABC touching AB, BC and AC at P, Q and R respectively.



Since, tangents drawn to a circle from an external point are equal,

$$AP = AR = 7\text{ cm}$$

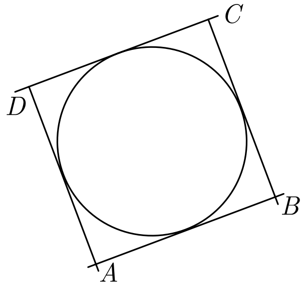
$$CQ = CR = 5\text{ cm}$$

Now, $BP = (AB - AP) = 10 - 7 = 3$ cm

$$BP = BQ = 3\text{ cm}$$

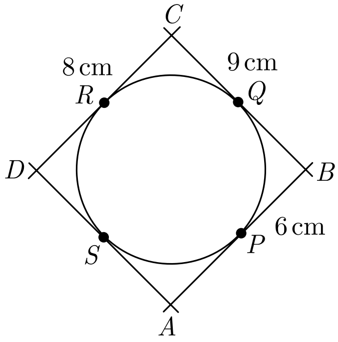
$$BC = BQ + QC = 3 + 5 = 8\text{ cm}$$

66. In figure, a circle touches all the four sides of a quadrilateral $ABCD$. If $AB = 6$ cm, $BC = 9$ cm and $CD = 8$ cm, then find the length of AD.



Ans : [Board 2020 Delhi Basic]

As per given information we have redrawn the figure below.



Tangents drawn from an external point to a circle are equal in length.

Thus $AP = AS$ and let it be x.

Similarly, $BP = BQ$, $CQ = CR$ and $RD = DS$

Now $BP = AB - AP = 6 - x$

$$BP = BQ = 6 - x$$

$$CQ = BC - BQ = 9 - (6 - x) = 3 + x$$

Now, $CQ = CR = 3 + x$

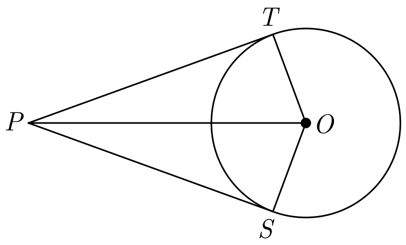
$$RD = CD - CR = 8 - (3 + x) = 5 - x$$

Now, $RD = DS = 5 - x$

$$AD = AS + SD = x + 5 - x = 5$$

Thus AD is 5 cm.

67. In the given figure, from a point P, two tangents PT and PS are drawn to a circle with centre O such that $\angle SPT = 120^\circ$, Prove that $OP = 2PS$.



Ans : [Board Term-2 Foreign 2016]

We have $\angle SPT = 120^\circ$

As OP bisects $\angle SPT$,

$$\angle OPS = \frac{120^\circ}{2} = 60^\circ$$

Since radius is always perpendicular to tangent,

$$\angle PTO = 90^\circ$$

Now in right triangle POS, we have

$$\cos 60^\circ = \frac{PS}{OP}$$

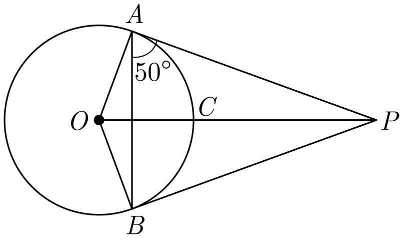
$$\frac{1}{2} = \frac{PS}{OP}$$

$$OP = 2PS$$ Hence proved.

68. From an external point P, tangents PA and PB are drawn to a circle with centre O. If $\angle PAB = 50^o$, then find $\angle AOB$.

Ans : [Board Term-2 Delhi 2016]

As per the given question we draw the figure as below.



Since $PA \perp OA$, $\angle OAP = 90^\circ$

$$\angle OAB = \angle OAP - \angle BAP$$

$$= 90^o - 50^o = 40^o$$

Since OA and OB are radii, we have

$$\angle OAB = \angle OBA = 40^o$$

Now

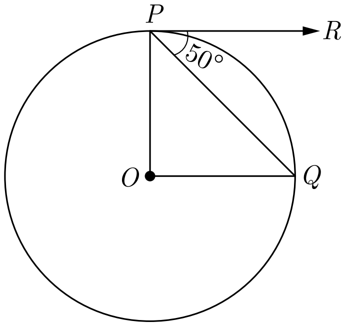
$$\angle AOB + \angle OAB + \angle OBA = 180^o$$

$$\angle AOB + 40^o + 40^o = 180^o$$

$$\angle AOB = 180^o - 80^o = 100^o$$

Hence $\angle AOB = 100^o$

69. If O is centre of a circle, PQ is a chord and the tangent PR at P makes an angle of 50° with PQ, find $\angle POQ$.



Ans : [Board Term-2, 2012]

We have $\angle RPQ = 50^\circ$

Since $\angle OPQ + \angle QPR$ is right angle triangle,

$$\angle OPQ = 90^\circ - 50^\circ = 40^\circ$$

Since, $OP = OQ$ because of radii of circle, we have

$$\angle OPQ = \angle OQR = 40^\circ$$

In ΔPOQ we have

$$\angle POQ = 180^\circ - (\angle OPQ + \angle OQP)$$

$$= 180^\circ - (40^\circ + 40^\circ)$$

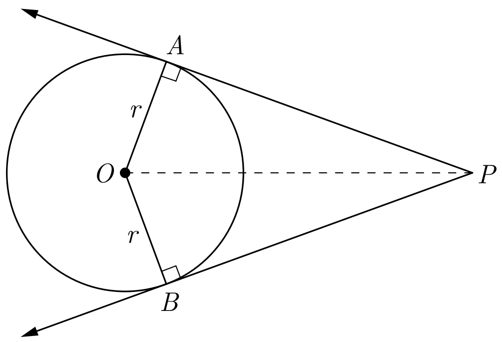
$$= 100^\circ$$

70. Prove that the lengths of two tangents drawn from an external point to a circle are equal.

Ans : [Board 2020 OD Basic, 2018]

Consider a circle of radius r and centre at O as shown in figure below. Here we have drawn two tangent from P at A and B. We have to prove that

$$AP = PB$$



We join OA, OB and OP. In ΔPAO and ΔPBO, OP

is common and $OA = OB$ radius of same circle.

Since radius is always perpendicular to tangent, at point of contact,

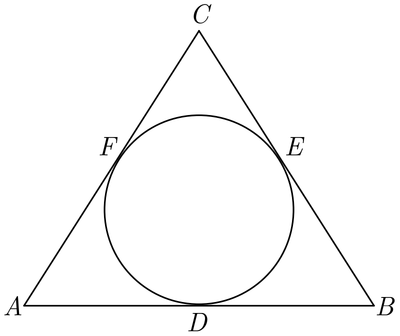
$$\angle OAP = \angle OBP = 90^\circ$$

Thus $\Delta PAO \cong \Delta PBO.$

and hence, $AP = BP$

Thus length of 2 tangents drawn from an external point to a circle are equal.

71. In the given figure, a circle is inscribed in a ΔABC, such that it touches the sides AB, BC and CA at points D, E and F respectively. If the lengths of sides AB, BC and CA are 12 cm, 8 cm and 10 cm respectively, find the lengths of AD, BE and CF.



Ans : [Board Term-2 Delhi 2016]

Since AF and AD are tangent of the circle, $AF = AD$

Let $AF = AD = x$

Now $DB = AB - AD = 12 - x$

Since BD and BE are tangent of the circle, $BD = BE$

Thus $BE = BD = 12 - x$

Now $CE = CB - BE = 8 - (12 - x)$

Since CF and CE are tangent of the circle, $CF = CE$

Thus $CF = CE = 8 - (12 - x)$ cm

But $AC = CF + FA$

Substituting values we have

$$10 = 8 - (12 - x) + x$$

$$10 = 2x - 4$$

$$2x = 10 + 4 = 14$$

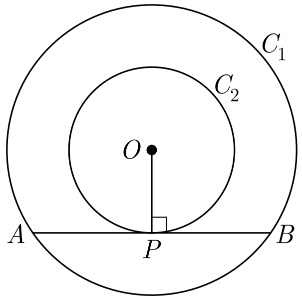
$$x = 7$$

Thus $AD = 7$ cm, $BE = 5$ cm, $CF = 3$ cm

72. Prove that in two concentric circles, the chord of the larger circle, which touches the smaller circle is bisected at the point of contact.

Ans : [Board Term-2, 2012]

As per the given question we draw the figure as below.

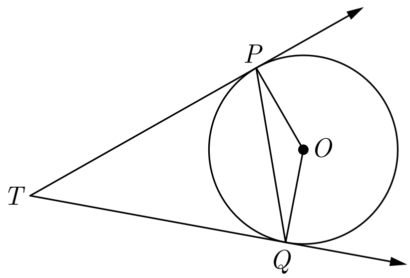


Since OP is radius and APB is tangent, $OP \perp AB$.

Now for bigger circle, O is centre and AB is chord such that $OP \perp AB$.

Thus OP bisects AB.

73. In the given figure PQ is chord of length 6 cm of the circle of radius 6 cm. TP and TQ are tangents to the circle at points P and Q respectively. Find $\angle PTQ$.



Ans : [Board Term-2 Delhi 2016]

We have $PQ = 6$ cm, $OP = OQ = 6$ cm

Since $PQ = OP = OQ$, triangle ΔPQO is an equilateral triangle.

Thus $\angle POQ = 60^\circ$

Now we know that $\angle POQ$ and $\angle PTQ$ are supplementary angle,

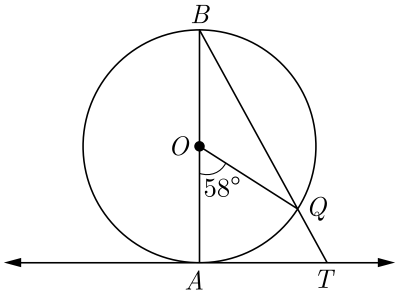
$$\angle POQ + \angle PTQ = 180^\circ$$

$$\angle PTQ = 180^\circ - \angle POQ$$

$$= 180^\circ - 60^\circ = 120^\circ$$

Thus $\angle PTQ = 120^\circ$

74. In given figure, AB is the diameter of a circle with centre O and AT is a tangent. If $\angle AOQ = 58^\circ$, find $\angle ATQ$.



Ans : [Board Term-2, 2015]

We have $\angle AOQ = 58^\circ$

Since angle $\angle ABQ$ and $\angle AOQ$ are the angle on the circumference of the circle by the same arc,

$$\angle ABQ = \frac{1}{2}\angle AOQ$$

$$= \frac{1}{2} \times 58^\circ = 29^\circ$$

Here OA is perpendicular to TA because OA is radius and TA is tangent at A.

Thus $\angle BAT = 90^\circ$

$$\angle ABQ = \angle ABT$$

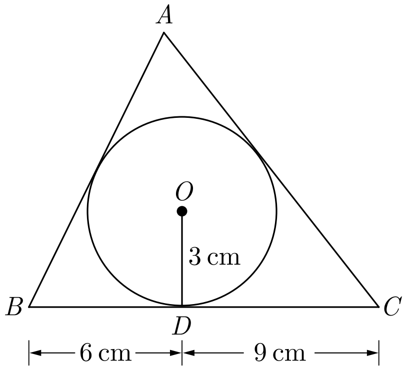
Now in ΔBAT,

$$\angle ATB = 90^\circ - \angle ABT$$

$$= 90^\circ - 29^\circ = 61^\circ$$

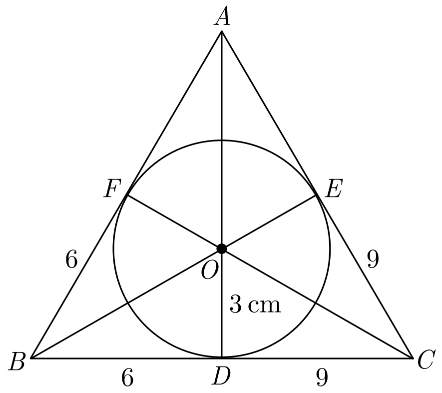
Thus $\angle ATQ = \angle ATB = 61^\circ$

75. In figure, a triangle ABC is drawn to circumscribe a circle of radius 3 cm, such that the segments BD and DC are respectively of lengths 6 cm and 9 cm. If the area of ΔABC is 54 cm^2, then find the lengths of sides AB and AC.



Ans : [Board Term-2 OD 2015]

We redraw the given circle as shown below.



Since tangents from an external point to a circle are equal,

$$AF = AE$$

$$BF = BD = 6 \text{ cm}$$

$$CE = CD = 9 \text{ cm}$$

Let $AF = AE = x$

Now $AB = AF + FB = 6 + x$

$$AC = AE + EC = x + 9$$

$$BC = 6 + 9 = 15 \text{ cm}$$

Perimeter of ΔABC,

$$p = 15 + 6 + x + 9 + x$$
$$= 30 + 2x$$

Now area, $\Delta ABC = \frac{1}{2} rp$

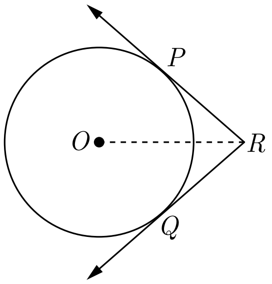
Here $r = 3$ is the radius of circle. Substituting all values we have

$$54 = \frac{1}{2} \times 3 \times (30 + 2x)$$
$$54 = 45 + 3x$$

or $x = 3$

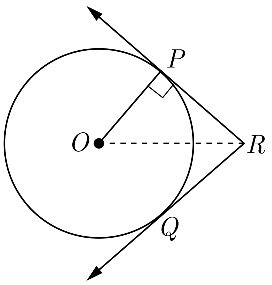
Thus $AB = 9$ cm, $AC = 12$ cm and $BC = 15$ cm.

76. In figure, two tangents RQ and RP are drawn from an external point R to the circle with centre O. If $\angle PRQ = 120^\circ$, then prove that $OR = PR + RQ$.



Ans : [Board Term-2 OD 2015]

We redraw the given figure by joining O to P as shown below.



$$\angle PRO = \frac{1}{2} \angle PRQ$$
$$= \frac{120^\circ}{2} = 60^\circ$$

Here ΔOPR is right angle triangle, thus

$$\angle POR = 90^\circ - \angle PRO$$
$$= 90^\circ - 60^\circ = 30^\circ$$

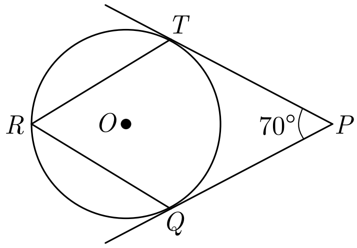
Now $$\frac{PR}{OR} = \sin 30^\circ = \frac{1}{2}$$

$$OR = 2PR = PR + PR$$

Since $PR = QR$,

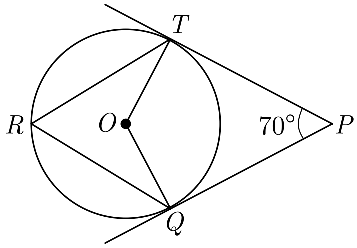
$$OR = PR + QR$$ Hence Proved

77. In figure, O is the centre of a circle. PT are tangents to the circle from an external point P. If $\angle TPQ = 70^\circ$, find $\angle TRQ$.



Ans : [Board Term-2 Foreign 2015]

We redraw the given figure by joining O to T and Q as shown below.



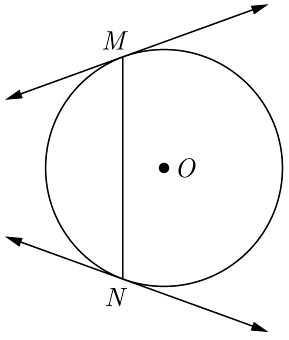
Here angle $\angle TOQ$ and $\angle TPQ$ are supplementary angle.

Thus $$\angle TOQ = 180^\circ - \angle TPQ$$
$$= 180^\circ - 70^\circ = 110^\circ$$

Since angle $\angle TRQ$ and $\angle TOQ$ are the angle on the circumference of the circle by the same arc,

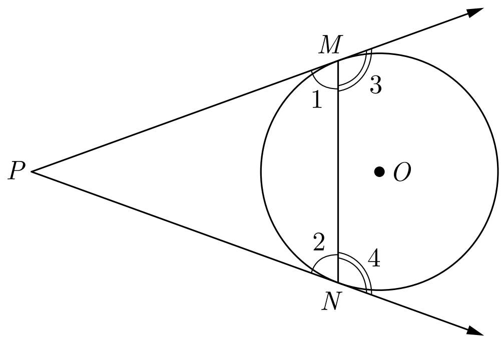
$$\angle TRQ = \frac{1}{2} \angle TOQ$$
$$= \frac{1}{2} \times 110^\circ = 55^\circ$$

78. Prove that tangents drawn at the ends of a chord of a circle make equal angles with the chord.



Ans : [Board Term-2 Delhi 2015]

We redraw the given figure by joining M and N to P as shown below.



Since length of tangents from an external point to a circle are equal,

$$PM = PN$$

Since angles opposite to equal sides are equal,

$$\angle 1 = \angle 2$$

Now using property of linear pair we have

$$180^{\circ} - \angle 1 = 180^{\circ} - \angle 2$$

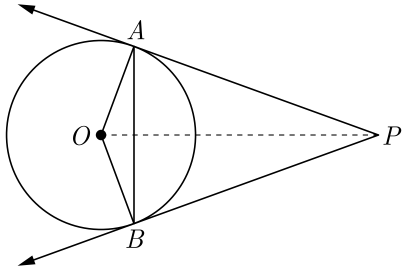
$$\angle 3 = \angle 4$$

Hence Proved

79. Two tangents PA and PB are drawn from an external point P to a circle inclined to each other at an angle of 70°, then what is the value of $\angle PAB$?

Ans : [Board Term-2, 2012]

As per question we draw the given circle as shown below.



Here angle $\angle AOB$ and $\angle APB$ are supplementary angle.

Thus $\angle AOB = 180^{\circ} - \angle APB$

$$= 180^{\circ} - 70^{\circ} = 110^{\circ}$$

OA and OB are radius of circle and equal in length, thus angle $\angle OAB$ and $\angle OBA$ are also equal. Thus in triangle ΔOAB we have

$$\angle OBA + \angle OAB + \angle AOB = 180^{\circ}$$

$$\angle OAB + \angle OBA = 180^{\circ} - \angle AOB$$

$$2\angle OAB = 180^{\circ} - 110^{\circ} = 70^{\circ}$$

$$\angle OAB = 35^{\circ}$$

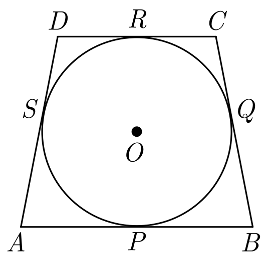
Since OA is radius and AP is tangent at A, $OA \perp AP$

$$\angle OAP = 90^{\circ}$$

Now $\angle PAB = \angle OAP - \angle OAB$

$$= 90^{\circ} - 35^{\circ} = 55^{\circ}$$

80. In Figure a quadrilateral $ABCD$ is drawn to circumscribe a circle, with centre O, in such a way that the sides AB, BC, CD, and DA touch the circle at the points P, Q, R and S respectively. Prove that. $AB + CD = BC + DA$.



Ans : [Board Term-2 OD 2016]

Since length of tangents from an external point to a circle are equal, At A, $AP = AS$ (1)

At B $BP = BQ$ (2)

At C $CR = CQ$ (3)

At D $DR = DS$ (4)

Adding eqn. (1), (2), (3), (4)

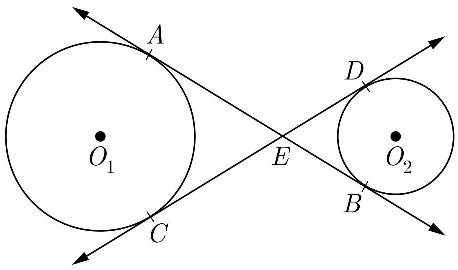
$$AP + BP + DR + CR = AS + DS + BQ + CQ$$

$$AP + BP + DR + RC = AS + SD + BQ + QC$$

$$AB + CD = AD + BC$$

Hence Proved

81. In Figure, common tangents AB and CD to the two circle with centres O_1 and O_2 intersect at E. Prove that $AB = CD$.



Ans : [Board Term-2 OD 2014]

Since EA and EC are tangents from point E to the circle with centre Q_1

$$EA = EC \quad ...(1)$$

and EB and ED are tangents from point E to the circle with centre O_2

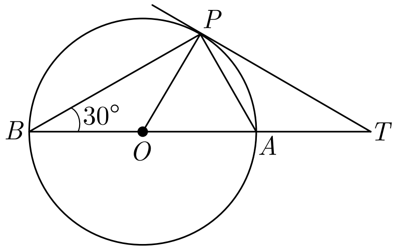
$$EB = ED \quad (2)$$

Adding eq (1) and (2) we have

$$EA + BE = CE + ED$$

$$AB = CD$$ Hence Proved

82. In the given figure, BOA is a diameter of a circle and the tangent at a point P meets BA when produced at T. If $\angle PBO = 30^o$, what is the measure of $\angle PTA$?



Ans : [Board Term-2, 2012]

Angle inscribed in a semicircle is always right angle.

$$\angle BPA = 90^o$$

Here OB and OP are radius of circle and equal in length, thus angle $\angle OBP$ and $\angle OPB$ are also equal.

Thus $\angle BPO = \angle PBO = 30^o$

Now $\angle POA = \angle OBP + \angle OPB$

$$= 30^\circ + 30^\circ = 60^\circ$$

Thus $\angle POT = \angle POA = 60^\circ$

Since OP is radius and PT is tangent at P, $OP \perp PT$

$$\angle OPT = 90^o$$

Now in right angle ΔOPT,

$$\angle PTO = 180^\circ - (\angle OPT + \angle POT)$$

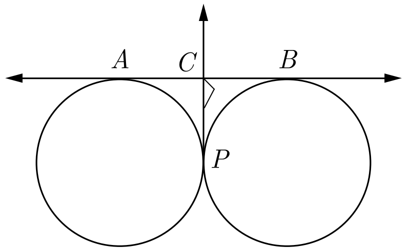
Substituting $\angle OPT = 90^o$ and $\angle POT = 60^\circ$ we have

$$\angle PTO = 180^\circ - (90^\circ + 60^\circ)$$

$$= 180^\circ - 150^\circ = 30^o$$

Thus $\angle PTA = \angle PTO = 30^o$

83. In the given figure, if $BC = 4.5$ cm, find the length of AB.



Ans : [Board Term-2, 2012]

Since length of tangents from an external point to a circle are equal,

$$CB = CP = 4.5 \text{ cm}$$

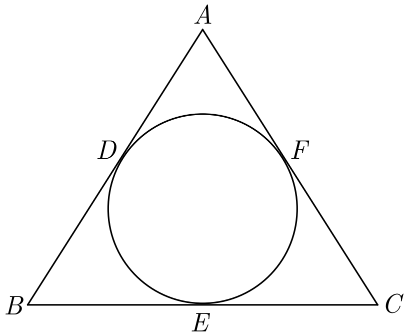
and $$CA = CP$$

Now $$AB = AC + CB$$

$$= CP + CP = 2CP$$

$$= 2 \times 4.5 = 9 \text{ cm}$$

84. In the given figure, if $AB = AC$, prove that $BE = CE$.



Ans : [Board Term-2 OD 2017]

Since tangents from an external point to a circle are equal,

$$AD = AF \quad (1)$$

$$BD = BE \quad (2)$$

$$CE = CF \quad (3)$$

From $AB = AC$ we have

$$AD + DB = AF + FC$$

or $$DB = FC \qquad (AD = AF)$$

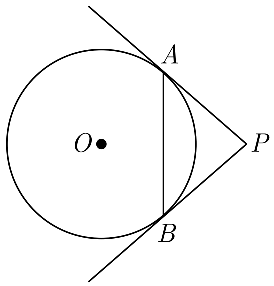
From eq (2) and (3) we have

$$BE = EC$$ Hence Proved

85. Prove that the tangents drawn at the end points of a chord of a circle make equal angles with the chord.

Ans : [Board Term-2 OD 2017]

As per question we draw figure shown below.



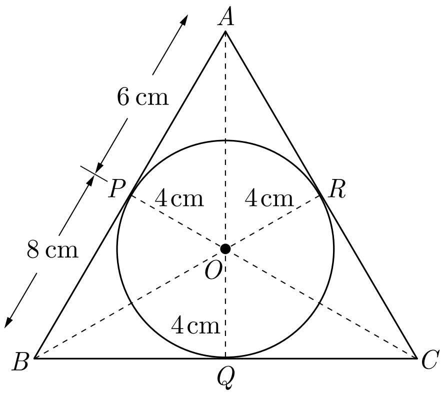
Since length of tangents from an external point to a circle are equal,

$$PA = PB$$

Since angles opposite to equal sides are equal,

$$\angle PAB = \angle PBA$$

86. In Figure the radius of incircle of ΔABC of area 84 cm^2 and the lengths of the segments AP and BP into which side AB is divided by the point of contact are 6 cm and 8 cm Find the lengths of the sides AC and BC.



Ans : [Board Term-2 Delhi 2012, 2014, OD Compt. 2017]

Since length of tangents from an external point to a circle are equal,

At A, $$AP = AR = 6 \text{ cm} \quad (1)$$

At B, $$,BP = BQ = 8 \text{ cm} \quad (2)$$

At C, $$CR = CQ = x \quad (3)$$

Perimeter of ΔABC,

$$p = AP + PB + BQ + QC + CR + RA$$

$$= 6 + 8 + 8 + x + x + 6$$

$$= 28 + 2x$$

Now area $$\Delta ABC = \tfrac{1}{2} rp$$

Here $r = 4$ is the radius of circle. Substituting all values we have

$$84 = \frac{1}{2} \times 4 \times (28 + 2x)$$

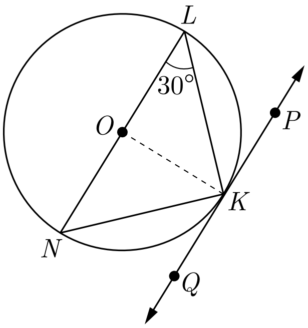
$$84 = 56 + 4x$$

$$21 = 14 + x \Rightarrow x = 7$$

Thus $AC = AR + RC = 6 + 7 = 13$ cm

$$BC = BQ + QC = 8 + 7 = 15 \text{ cm}$$

87. In figure, O is the centre of the circle and LN is a diameter. If PQ is a tangent to the circle at K and $\angle KLN = 30^{\circ}$, find $\angle PKL$.



Ans : [Board Term-2 OD Compt 2017]

Since OK and OL are radius of circle, thus

$$OK = OL$$

Angles opposite to equal sides are equal,

$$\angle OKL = \angle OLK = 30^{\circ}$$

Tangent is perpendicular to the end point of radius,

$$\angle OKP = 90^{\circ} \qquad \text{(Tangent)}$$

Now $\angle PKL = \angle OKP - \angle OKL$

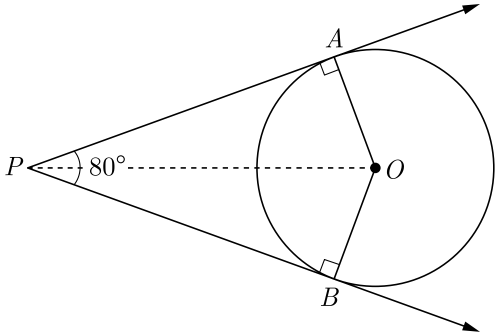
$$= 90^{\circ} - 30^{\circ} = 60^{\circ}$$

THREE MARKS QUESTIONS

88. If tangents PA and PB drawn from an external point P to a circle with centre O are inclined to each other at an angle of 80°, then find $\angle POA$.

Ans : [Board 2020 Delhi Basic]

As per given information we have drawn the figure below.



Since PA and PB are the tangents, PO will be angle bisector of $\angle P$

Hence, $\angle APO = 40^{\circ}$

Now, in ΔAPO, $\angle PAO$ is 90° because this is angle between radius and tangent.

Now $\angle PAO + \angle APO + \angle POA = 180^{\circ}$

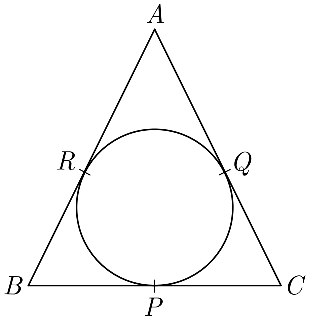
$$90^{\circ} + 40^{\circ} + \angle POA = 180^{\circ}$$

$$\angle POA = 50^{\circ}$$

89. An isosceles triangle ABC, with $AB = AC$, circumscribes a circle, touching BC at P, AC at Q and AB at R. Prove that the contact point P bisects BC.

Ans : [Board 2020 OD Basic]

As per given information we have drawn the figure below.



Since, the tangents drawn from externals points are equal,

$$AR = AQ$$

$$BR = BP$$

$$CP = CQ$$

Now we have, $AB = AC$

$$AR + BR = AQ + CQ$$

$$AR + BP = AQ + CP$$

$$AQ + BP = AQ + CP$$

$$BP = CP$$

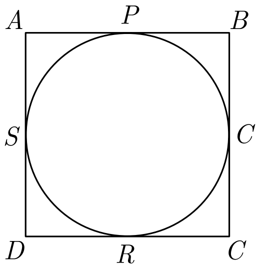
Hence, the point of contact P bisects BC.

90. Prove that the rectangle circumscribing a circle is a square.

Ans : [Board 2020 SQP Standard]

We have a rectangle $ABCD$ circumscribe a circle which touches the circle at P, Q, R, S. We have to prove that $ABCD$ is a square.

As per given information we have drawn the figure below.



Since tangent drawn from an external point to a circle are equals,

$$AP = AS$$
$$PB = BQ$$
$$DR = DS$$
$$RC = QC$$

Adding all above equation we have

$$AP + PB + DR + RC = AS + SD + BQ + QC$$
$$AB + CD = AD + BC$$

Since $ABCD$ is rectangle, $AB = CD$ and $AD = BC$,

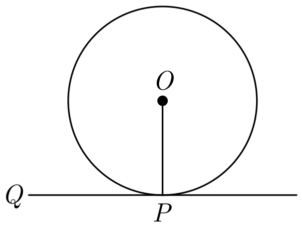
Thus $$2AB = 2BC$$
$$AB = BC$$

Since adjacent sides are equal are equal. So, $ABCD$ is a square.

91. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

Ans : [Board 2020 Delhi Basic]

Given, a circle with centre O and tangent AB at P. We take a point Q on the tangent AB and join OQ meeting the circle at R.



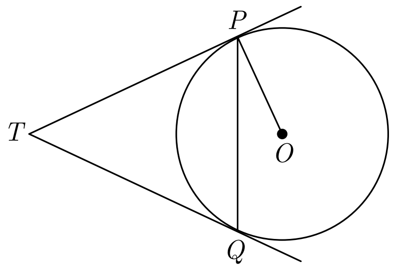
To prove that $OP \perp AB$, it is sufficient to prove that OP is shorter than any other segment joining O to any point of AB.

Clearly $$OP = OR \quad \text{(radius)}$$
$$OQ = OR + RQ$$
$$OQ > OR$$
$$OQ > OP$$

Thus OP is shorter than any other segment joining O to any other point of AB and shortest line is perpendicular.

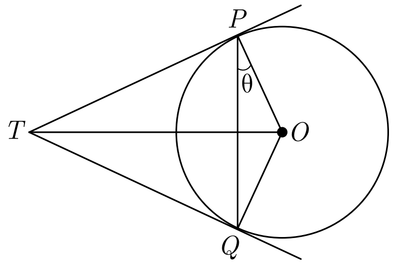
Thus $$OP \perp AB$$ Hence Proved

92. In figure, two tangents TP and TQ are drawn to circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$.



Ans : [Board 2020 Delhi Standard]

We redraw the given figure as shown below.



Let $\angle OPQ$ be θ, then

$$\angle TPQ = 90^\circ - \theta$$

Since, $TP = TQ$, due to opposite angles of equal sides we have

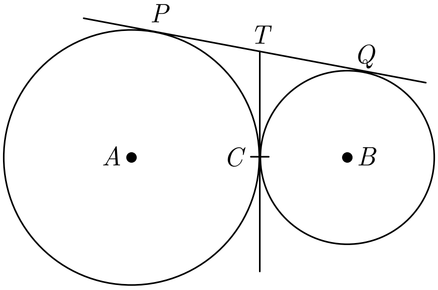
$$\angle TQP = 90^\circ - \theta$$

From angle sum property of a triangle we can write,

$$\angle TPQ + \angle TQP + \angle PTQ = 180^\circ$$
$$90^\circ - \theta + 90^\circ - \theta + \angle PTQ = 180^\circ$$
$$\angle PTQ = 180^\circ - 180^\circ + 2\theta$$
$$\angle PTQ = 2\theta$$

Hence, $\angle PTQ = 2\angle OPQ$

93. In given figure, two circles touch each other at the point C. Prove that the common tangent to the circles at C, bisects the common tangent at P and Q.



Ans : [Board 2020 OD Basic, 2020 Delhi Standard]

Here PT and TC are the tangents of circle A from extended point, thus

$$PT = TC$$

Here TQ and TC are the tangents of circle B from extended point, thus

$$QT = TC$$

Thus, $PT = QT$

Now, $PQ = PT + TQ$
$= PT + PT$
$= 2PT$

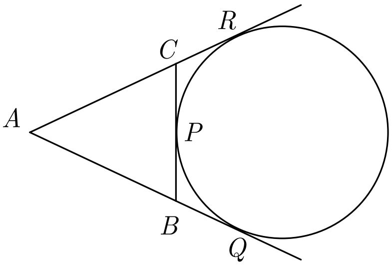
Thus $\frac{1}{2}PQ = PT$

Hence, the common tangent to the circle at C, bisects the common tangents at P and Q.

94. If a circle touches the side BC of a triangle ABC at P and extended sides AB and AC at Q and R, respectively, prove that $AQ = \frac{1}{2}(BC + CA + AB)$

Ans : [Board 2020 OD Standard, 2016]

As per given information in question we have drawn the figure below,



From the same external point, the tangent segments drawn to a circle are equal.

From the point B, $BQ = BP$

From the point A, $AQ = AR$

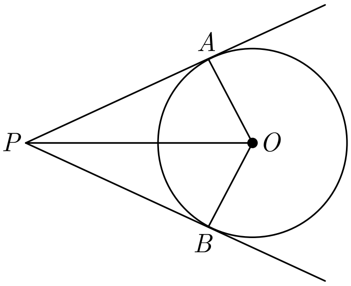
From the point C, $CP = CR$

Now

$$\begin{aligned} AB + BC + CA &= (AQ - BQ) + (BP + PC) + (AR - CR) \\ &= (AQ - BQ) + (BQ + CR) + (AQ - CR) \\ &= 2AQ \end{aligned}$$

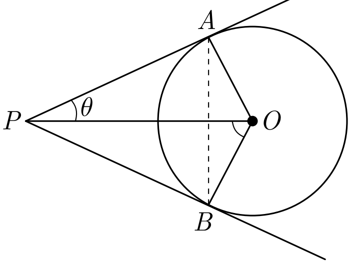
$$AQ = \frac{1}{2}(BC + CA + AB)$$ Hence proved.

95. In the given figure, OP is equal to the diameter of a circle with centre O and PA and PB are tangents. Prove that ABP is an equilateral triangle.



Ans : [Board Term-2, 2014]

We redraw the given figure by joining A to B as shown below.



Since OA is radius and PA is tangent at A, $OA \perp AP$.

Now in right angle triangle ΔOAP, OP is equal to diameter of circle, thus

$$OP = 2OA$$

$$\frac{OA}{OP} = \frac{1}{2}$$

$$\sin\theta = \frac{1}{2} \Rightarrow \theta = 30^\circ$$

Since PO bisect the angle $\angle APB$,

Hence, $\angle APB = 2 \times 30^\circ = 60^o$

Now, in ΔAPB,

$$AP = AB$$

$$\angle PAB = \angle PBA$$

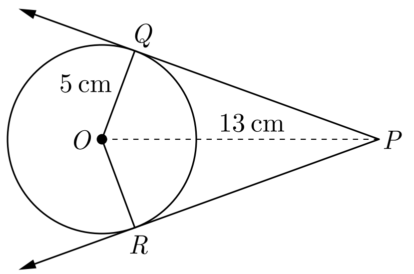
$$= \frac{180^\circ - 60^\circ}{2} = 60^o$$

Thus ΔAPB is an equilateral triangle.

96. From a point P, which is at a distant of 13 cm from the centre O of a circle of radius 5 cm, the pair of tangents PQ and PR are drawn to the circle, then the area of the quadrilateral $PQOR$ (in cm^2).

Ans : [Board Term-2, 2012]

As per the given question we draw the figure as below.



Here OQ is radius and QP is tangent at Q, since radius is always perpendicular to tangent at point of contact, ΔOQP is right angle triangle.

Now $PQ = \sqrt{OP^2 - OR^2}$

$$= \sqrt{13^2 - 5^2}$$

$$= \sqrt{169 - 25}$$

$$= \sqrt{144} = 12 \text{ cm}$$

Area of triangle ΔOQP,

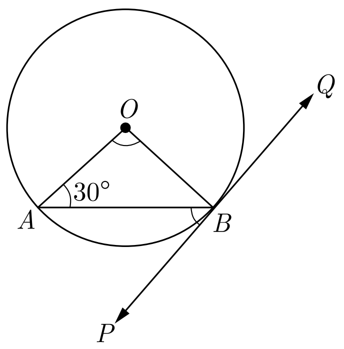
$$\Delta = \tfrac{1}{2}(OQ)(QP)$$

$$= \tfrac{1}{2} \times 12 \times 5 = 30$$

Area of quadrilateral $PQOR$,

$$2 \times \Delta POQ = 2 \times 30 = 60 \text{ cm}^2$$

97. In the figure, PQ is a tangent to a circle with centre O. If $\angle OAB = 30^o$, find $\angle ABP$ and $\angle AOB$.



Ans : [Board Term-2 Delhi 2014]

Here OB is radius and QT is tangent at B, $OB \perp PQ$

$$\angle OBP = 90^o$$

Here OA and OB are radius of circle and equal. Since angles opposite to equal sides are equal,

$$\angle OAB = \angle OBA = 30^o$$

Now $\angle AOB = 180^o - (30^o + 30^o$

$$= 120^o$$

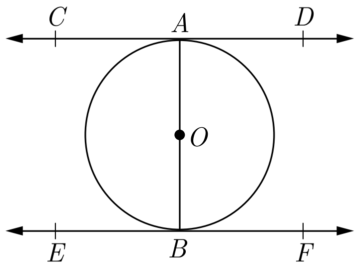
$$\angle ABP = \angle OBP - \angle OBA$$

$$= 90^o - 30^o = 60^o$$

98. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Ans : [Board 2020 Delhi Basic, 2017, 2014]

Let AB be a diameter of a given circle and let CD and RF be the tangents drawn to the circle at A and B respectively as shown in figure below.



Here $AB \perp CD$ and $AB \perp EF$

Thus $\angle CAB = 90^o$ and $\angle ABF = 90^o$

Hence $\angle CAB = \angle ABF$

and $\angle ABE = \angle BAD$

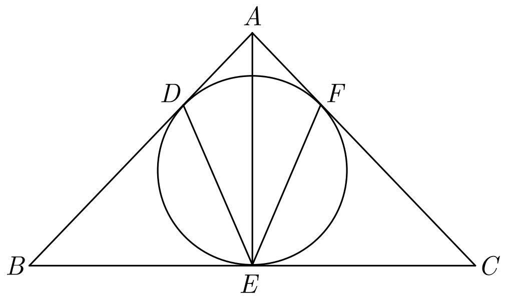
Hence $\angle CAB$ and $\angle ABF$ also $\angle ABE$ and $\angle BAD$ are alternate interior angles.

$CD || EF$ Hence Proved

99. In $\Delta ABD, AB = AC$. If the interior circle of ΔABC touches the sides AB, BC and CA at D, E and F respectively. Prove that E bisects BC.

Ans : [Board Term-2 Delhi 2014, 2012]

As per question we draw figure shown below.



Since length of tangents from an external point to a circle are equal,

At A, $AF = AD$ (1)

At B $BE = BD$ (2)

At C $CE = CF$ (3)

Now we have $AB = AC$

$$AD + DB = AF + FC$$

$$BD = FC \qquad (AD = AF)$$

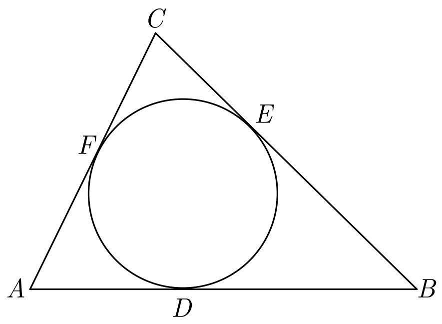
$$BE = EC \qquad (BD = BE, CE = CF)$$

Thus E bisects BC.

100. A circle is inscribed in a ΔABC, with sides AC, AB and BC as 8 cm, 10 cm and 12 cm respectively. Find the length of AD, BE and CF.

Ans : [Board Term-2 Delhi 2013, 2012]

As per question we draw figure shown below.



We have $AC = 8$ cm

$AB = 10$ cm

and $BC = 12$ cm

Let AF be x. Since length of tangents from an external point to a circle are equal,

At A, $AF = AD = x$ (1)

At B $BE = BD = AB - AD = 10 - x$ (2)

At C $CE = CF = AC - AF = 8 - x$ (3)

Now $BC = BE + EC$

$$12 = 10 - x + 8 - x$$

$$2x = 18 - 12 = 6$$

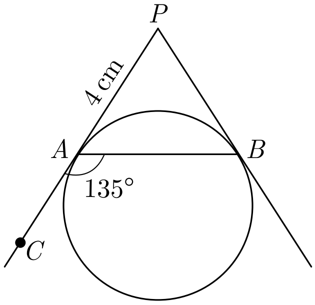
or $x = 3$

Now $AD = 3$ cm,

$BE = 10 - 3 = 7$ cm

and $CF = 8 - 3 = 5$

101. In the given figure, PA and PB are tangents to a circle from an external point P such that $PA = 4$ cm and $\angle BAC = 135^0$. Find the length of chord AB.



Ans : [Board Term-2 OD 2017]

Since length of tangents from an external point to a circle are equal,

$$PA = PB = 4 \text{ cm}$$

Here $\angle PAB$ and $\angle BAC$ are supplementary angles,

$$\angle PAB = 180^\circ - 135^\circ = 45^\circ$$

Angle $\angle ABP$ and $= \angle PAB = 45^\circ$ opposite angles of equal sides, thus

$$\angle ABP = \angle PAB = 45^\circ$$

In triangle ΔAPB we have

$$\angle APB$$

$$= 180^\circ - \angle ABP - \angle BAP$$

$$= 180^\circ - 45^\circ - 45^\circ = 90^\circ$$

Thus ΔAPB is a isosceles right angled triangle

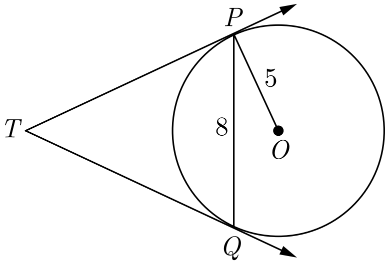
Now $\qquad AB^2 = AP^2 + BP^2 = 2AP^2$

$$= 2 \times 4^2 = 32$$

Hence $\qquad AB = \sqrt{32} = 4\sqrt{2}$ cm

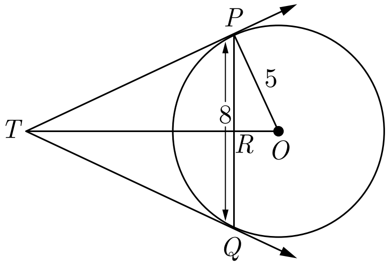
FOUR MARKS QUESTIONS

102. In Figure, PQ is a chord of length 8 cm of a circle of radius 5 cm and centre O. The tangents at P and Q intersect at point T. Find the length of TP.



Ans : [Board 2019 Delhi Standard]

We redraw the given figure as shown below.
Here OT is perpendicular bisector of PQ,



Since, OT is perpendicular bisector of PQ,

, $\qquad PR = QR = 4 \text{ cm}$

In right angle triangle ΔOTP and ΔPTR, we have

$$TP^2 = TR^2 + PR^2 \qquad \text{...(1)}$$

Also, $\qquad OT^2 = TP^2 + OP^2$

Substituting TP^2 from equation (1) we have

$$OT^2 = (TR^2 + PR^2) + OP^2$$

$$(TR + OR)^2 = TR^2 + PR^2 + OR^2$$

Now $\qquad OR^2 = OP^2 - PR^2$

$$= 5^2 - 4^2 = 3^2$$

Thus $\qquad OR = 3$ cm

Thus substituting $OR = 3$ cm we have

$$(TR + 3)^2 = TR^2 + 4^2 + 5^2$$

$$TR^2 + 9 + 6TR = TR^2 + 16 + 25$$

$$6TR = 32$$

$$TR = \frac{16}{3}$$

Now, from (1), $\quad TP^2 = TR^2 + PR^2$

$$= \left(\frac{16}{3}\right)^2 + 4^2$$

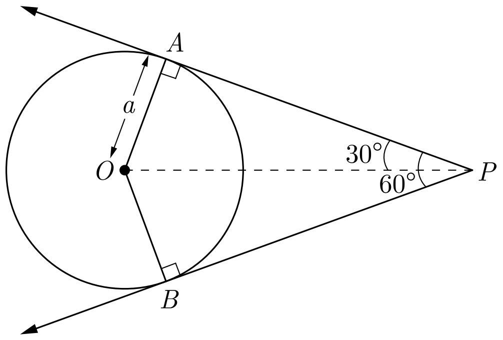
$$= \frac{256}{9} + 16 = \frac{400}{9}$$

$$TP = \frac{20}{3} \text{ cm}$$

103. If the angle between two tangents drawn from an external point P to a circle of radius a and centre O, is 60^o, then find the length of OP.

Ans : [Board 2020 SQP STD]

As per the given question we draw the figure as below.



Tangents are always equally inclined to line joining the external point P to centre O.

$$\angle APO = \angle BPO = \frac{60^{\circ}}{2} = 30^{\circ}$$

Also radius is also perpendicular to tangent at point of contact.

In right $\Delta\, OAP$ we have,

$$\angle APO = 30^{\circ}$$

Now, $$\sin 30^{\circ} = \frac{OA}{OP}$$

Here OA is radius whose length is a, thus

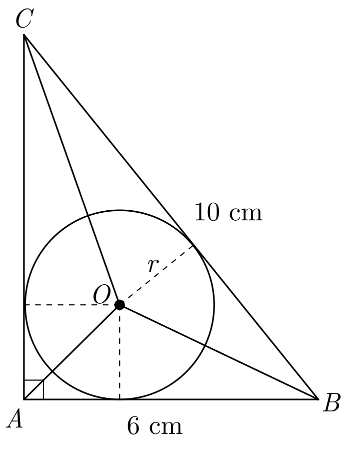
$$\frac{1}{2} = \frac{a}{OP}$$

or $$OP = 2a$$

104. A right triangle ABC, right angled at A is circumscribing a circle. If $AB = 6$ cm and $BC = 10$ cm, find the radius r of the circle.

Ans : [Board 2020 Delhi Basic]

As per question we draw figure shown below.



In triangle $\Delta\, ABC$,

$$AC = \sqrt{10^2 - 36^2} = 8 \text{ cm}$$

Area of triangle $\Delta\, ABC$,

$$\Delta\, ABC = \frac{1}{2} \times AB \times AC$$

$$= \frac{1}{2} \times 6 \times 8 = 24 \text{ cm}^2$$

Here we have joined AO, BO and CO.

For area of triangle we have

$$\Delta\, ABC = \Delta\, OBC + \Delta\, OCA + \Delta\, OAB$$

$$24 = \tfrac{1}{2} rBC + \tfrac{1}{2} rAC + \tfrac{1}{2} rAB$$

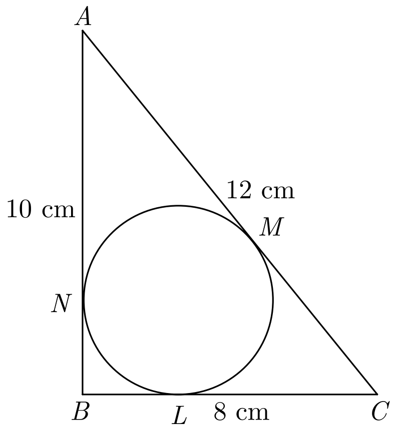
$$= \tfrac{1}{2} r(BC + AC + AB)$$

$$= \tfrac{1}{2} r(6 + 10 + 8) = 12r$$

or $$12r = 24$$

Thus $r = 2$ cm.

105. In figure, a circle is inscribed in a $\Delta\, ABC$ having sides $BC = 8$ cm, $AB = 10$ cm and $AC = 12$ cm. Find the length BL, CM and AN.



Ans : [Board 2019 Delhi Standard]

Tangents from external a point on a circle are always equal in length.

Let x be length of BL, then we have

$$BL = x = BN$$

So, $$LC = MC = (8 - x)$$

and $$AN = AM = (10 - x)$$

Since, $$AC = 12$$

$$AM + MC = 12$$

$$(10 - x) + (8 - x) = 12$$

$$18 - 2x = 12 \Rightarrow x = 3$$

Hence, $BL = 3\,\text{cm}$

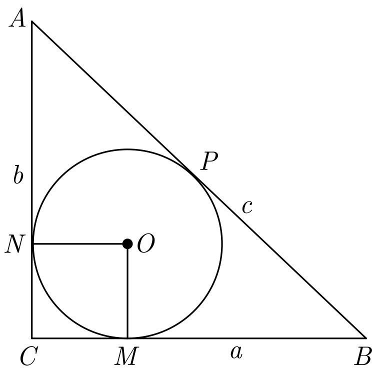
$CM = 8 - 3 = 5\,\text{cm}$

and $AN = 10 - 3 = 7\,\text{cm}$

106. a, b and c are the sides of a right triangle, where c is the hypotenuse. A circle, of radius r, touches the sides of the triangle. Prove that $r = \frac{a+b-c}{2}$.

Ans : [Board Term-2 Delhi 2016]

As per question we draw figure shown below.



Let the circle touches CB at M, CA at N and AB at P.

Now $OM \perp CB$ and $ON \perp AC$ because radius is always perpendicular to tangent

OM and ON are radius of circle, thus

$$OM = ON$$

CM and CN are tangent from C, thus

$$CM = CN$$

Therefore $OMCN$ is a square. Let

Let $OM = r = CM = CN = ON$

Since length of tangents from an external point to a circle are equal,

$$AN = AP, CN = CM \text{ and } BM = BP$$

Now taking $AN = AP$

$$AC - CN = AB - BP$$

$$b - r = c - BM$$

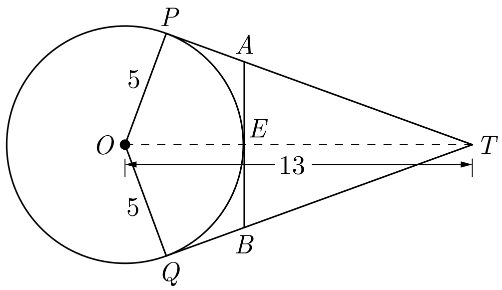
$$b - r = c - (a - r)$$

$$b - r = c - a + r$$

$$2r = a + b - c$$

$$r = \frac{a+b-c}{2}$$ Hence Proved.

107. In figure O is the centre of a circle of radius 5 cm. T is a point such that $OT = 13$ cm and OT intersects circle at E. If AB is a tangent to the circle at E, find the length of AB, where TP and TQ are two tangents to the circle.



Ans : [Board Term-2 Delhi 2016]

Here ΔOPT is right angled triangle because PT is tangent on radius OP.

Thus $PT = \sqrt{13^2 - 5^2}$

$$= \sqrt{169 - 25} = 12 \text{ cm}$$

and $TE = OT - OE$

$$= 13 - 5 = 8 \text{ cm}$$

Since length of tangents from an external point to a circle are equal,

Let $PA = AE = x$

Here ΔAET is right angled triangle because AB is tangent on radius OE.

In ΔAET, $TA^2 = TE^2 + EA^2$

$$(TP - PA)^2 = 8^2 + x^2$$

$$(12 - x)^2 = 64 + x^2$$

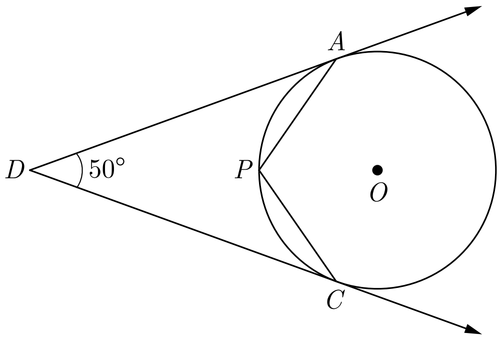
$$144 - 24x + x^2 = 64 + x^2$$

$$24x = 144 - 64 = 80$$

or, $x = 3.3$ cm.

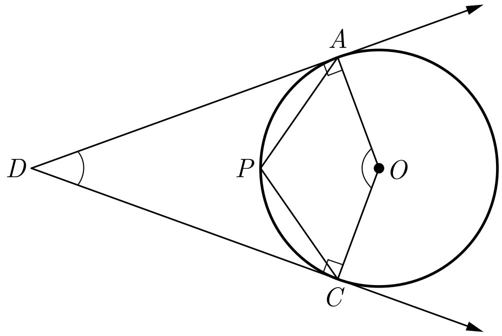
Thus $AB = 2 \times x = 2 \times 3.3 = 6.6$ cm.

108. In the given figure, O is the centre of the circle. Determine $\angle APC$, if DA and DC are tangents and $\angle ADC = 50^\circ$.



Ans : [Board Term-2, 2015]

We redraw the given figure by joining A and C to O as shown below.



Since DA and DC are tangents from point D to the circle with centre O, and radius is always perpendicular to tangent, thus

$$\angle DAO = \angle DCO = 90^\circ$$

and

$$\angle ADC + \angle DAO + \angle DCO + \angle AOC = 360^\circ$$

$$50^\circ + 90^\circ + 90^\circ + \angle AOC = 360^\circ$$

$$230^\circ + \angle AOC = 360^\circ$$

$$\angle AOC = 360^\circ - 230^\circ = 130^\circ$$

Now Reflex $\angle AOC = 360^\circ - 130^\circ = 230^\circ$

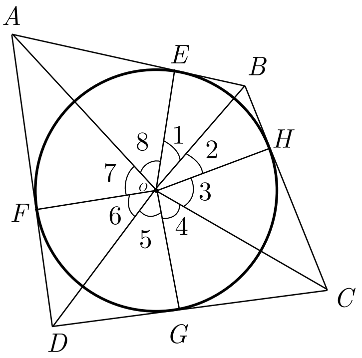
$$\angle APC = \frac{1}{2} \text{ reflex } \angle AOC$$

$$= \frac{1}{2} \times 230^\circ = 115^\circ$$

109. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Ans : [Board Term-2 Foreign 2017,

A circle centre O is inscribed in a quadrilateral as shown in figure given below.



Since OE and OF are radius of circle,

$$OE = OF$$

Tangent drawn at any point of a circle is perpendicular to the radius through the point contact.

Thus $\angle OEA = \angle OFA = 90^\circ$

Now in ΔAEO and ΔAFO,

$$OE = OF$$

$$\angle OEA = \angle OFA = 90^\circ$$

$$OA = OA \quad \text{(Common side)}$$

Thus $\Delta AEO \cong \Delta AFO$ (SAS congruency)

$$\angle 7 = \angle 8$$

Similarly, $\angle 1 = \angle 2$

$$\angle 3 = \angle 4$$

$$\angle 5 = \angle 6$$

Since angle around a point is 360°,

$$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^\circ$$

$$2\angle 1 + 2\angle 8 + 2\angle 4 + 2\angle 5 = 360^\circ$$

$$\angle 1 + \angle 8 + \angle 4 + \angle 5 = 180^\circ$$

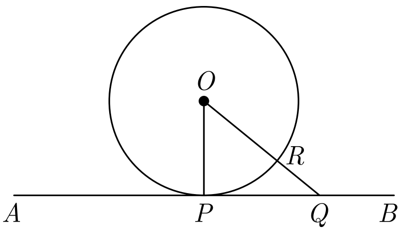
$$(\angle 1 + \angle 8) + (\angle 4 + \angle 5) = 180^\circ$$

$$\angle AOB + \angle COD = 180^\circ$$ Hence Proved.

110. Prove that tangent drawn at any point of a circle perpendicular to the radius through the point contact.

Ans : [Board Term-2 OD 2016]

Consider a circle with centre O with tangent AB at point of contact P as shown in figure below



Let Q be point on AB and we join OQ. Suppose it touch the circle at R.

We $\quad OP = OR \quad$ (Radius)

Clearly $\quad OQ > OR$

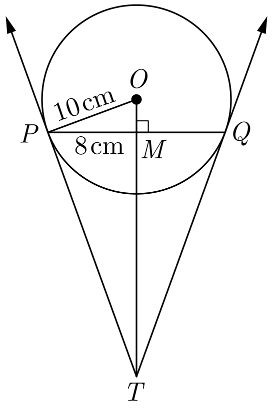
$$OQ > OP$$

Same will be the case with all other points on circle. Hence OP is the smallest line that connect AB and smallest line is perpendicular.

Thus $\quad OP \perp AB$

or, $\quad OP \perp PQ \quad$ Hence Proved

111. In figure, PQ, is a chord of length 16 cm, of a circle of radius 10 cm. the tangents at P and Q intersect at a point T. Find the length of TP.



Ans : [Board Term-2 Delhi 2014]

Here PQ is chord of circle and OM will be perpendicular on it and it bisect PQ. Thus $\Delta\,OMP$ is a right angled triangle.

We have $\quad OP = 10$ cm $\quad$ (Radius)

$PM = 8$ cm $\quad (PQ = 16$ cm$)$

Now in $\Delta\,OMP, OM = \sqrt{10^2 - 8^2}$

$$= \sqrt{100 - 64} = \sqrt{36}$$

$$= 6 \text{ cm}$$

Now $\quad \angle TPM + \angle MPO = 90^\circ$

Also, $\quad \angle TPM + \angle PTM = 90^\circ$

$$\angle MPO = \angle PTM$$

$$\angle TMP = \angle OMP = 90^\circ$$

Thus due to AA symmetry we have

$$\Delta\,TMP \sim \Delta\,PMO$$

Now $\quad \dfrac{TP}{PO} = \dfrac{MP}{MO}$

$$\frac{TP}{10} = \frac{8}{6}$$

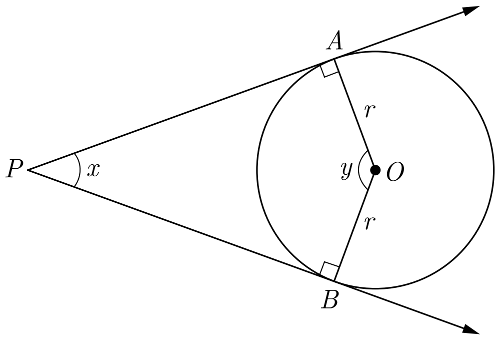
$$TP = \frac{80}{6} = \frac{40}{3}$$

Hence length of TP is $\frac{40}{3}$ cm.

112. Two tangents PA and PB are drawn from an external point P to a circle with centre O, such that $\angle APB = \angle x$ and $\angle AOB = y$. Prove that opposite angles are supplementary.

Ans : [Board Term-2, 2011]

As per question we draw figure shown below.



Now $OA \perp AP$ and $OB \perp BP$ because tangent drawn at any point of a circle is perpendicular to the radius through the point contact.

Thus $\quad \angle A = \angle B = 90^\circ$

Since, $AOBP$ is a quadrilateral,

$$\angle A + \angle B + x + y = 360^\circ$$

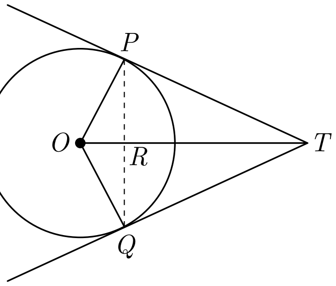
$$90^\circ + 90^\circ + x + y = 360^\circ$$

$$180 + x + y = 360^\circ$$

$$x + y = 180^\circ$$

Therefore opposite angle are supplementary.

113. In figure PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents drawn at P and Q intersect at T. Find the length of TP.



Ans : [Board Term-2 OD Compt 2017]

Since length of tangents from an external point to a circle are equal,

$$PT = QT$$

Thus ΔTPQ is an isosceles triangle and TO is the angle bisector of $\angle PTQ$.

Thus $OT \perp PQ$ and OT also bisects PQ.

Thus $$PR = PQ = \tfrac{8}{2} = 4 \text{ cm}$$

Since $\Delta\, OPR$ is right angled isosceles triangle,

$$OR = \sqrt{OP^2 - PR^2}$$

$$= \sqrt{5^2 - 4^2} = \sqrt{25 - 16}$$

$$= 3 \text{ cm}$$

Now, Let $TP = x$ and $TR = y$ then we have

$$x^2 = y^2 + 16 \qquad (1)$$

Also in $\Delta\, OPT$,

$$x^2 + (5)^2 = (y+3)^2 \qquad (2)$$

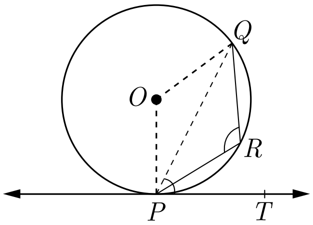
Solving (1) and (2) we get

$$y = \frac{16}{3} \text{ and } x = \frac{20}{3}$$

Hence, $TP = \dfrac{20}{3}$

114. In figure, PQ is a chord of a circle O and PT is a tangent. If $\angle QPT = 60^\circ$, find $\angle PRQ$.

Ans : [Board Term-2 OD 2015, 2017]



We have $\angle QPT = 60^\circ$

Here $\angle OPT = 90^\circ$ because of tangent at radius.

Now $$\angle OPQ = \angle OQP$$

$$= \angle OPT - \angle QTP$$

$$= 90^\circ - 60^\circ = 30^\circ$$

$$\angle POQ = 180^\circ - (\angle OPQ + \angle OQP)$$

$$= 180^\circ - (30^\circ + 30^\circ)$$

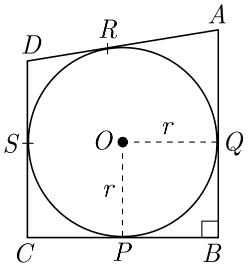
$$= 180^\circ - 60^\circ = 120^\circ$$

Now Reflex $\angle POQ = 360^\circ - 120^\circ = 240^\circ$

$$\angle PRQ = \frac{1}{2} \text{ Reflex } \angle POQ$$

$$= \frac{1}{2} \times 240^\circ = 120^\circ$$

115. In figure, a circle with centre O is inscribed in a quadrilateral $ABCD$ such that, it touches the sides BC, AB, AD and CD at points P, Q, R and S respectively. If $AB = 29$ cm, $AD = 23$ cm, $\angle B = 90^\circ$ and $DS = 5$ cm, then find the radius of the circle (in cm).



Ans : [Board Term-2, 2013]

Since length of tangents from an external point to a circle are equal,

$$DR = DS = 5 \text{ cm}$$

$$AR = AQ$$

$$BQ = BP$$

Now $$AR = AD - DR$$

$$= 23 - 5 = 18 \text{ cm}$$

$$AQ = AR = 18 \text{ cm}$$

$$QB = AB - AQ$$

$$= 29 - 18 = 11 \text{ cm}$$

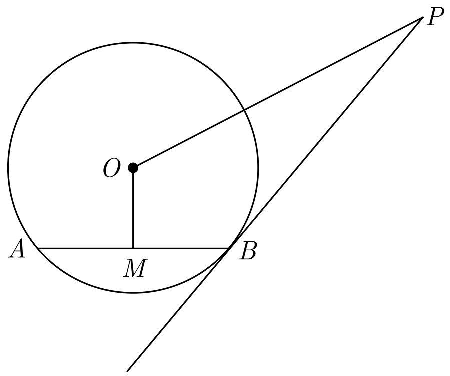
$$PB = QB = 11$$

Now $\angle OQB = \angle OPB = 90^\circ$ because radius is always perpendicular to tangent.

Thus $$OP = OQ = PB = BQ$$

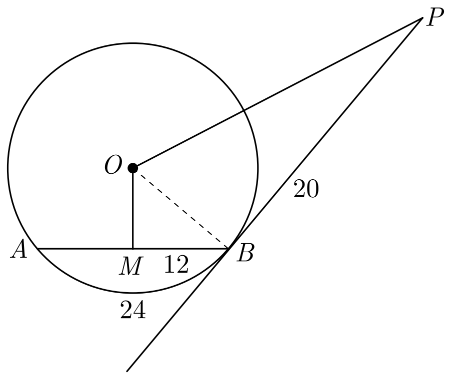
So, $POQB$ is a square. Hence, $r = OP = PB = 11$ cm

116. PB is a tangent to the circle with centre O to $B.AB$ is a chord of length 24 cm at a distance of 5 cm from the centre. It the tangent is length 20 cm, find the length of PO.



Ans : [Board Term-2 Delhi 2015]

We redraw the given figure by joining O to B as shown below.



Here $\Delta\, OMB$ right angled triangle because AB is chord and OM is perpendicular on it.

In right angled triangle $\Delta\, OMB$ we have,

$$OB^2 = OM^2 + MB^2$$

$$= 5^2 + 12^2 = 13^2$$

Thus $$OB = 13$$

Here $\Delta\, OBP$ right angled triangle because PB is tangent on radius OB.

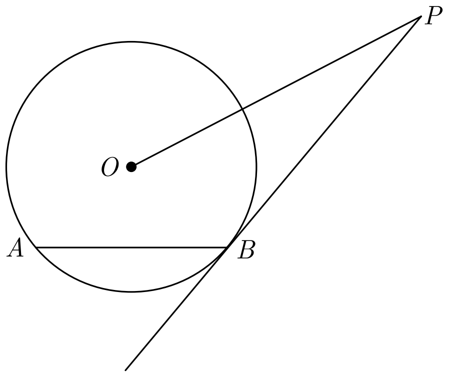
This in right angled triangle $\Delta\, OBP$ we have,

$$OP^2 = OB^2 + BP^2$$

$$= 13^2 + 20^2 = 569$$

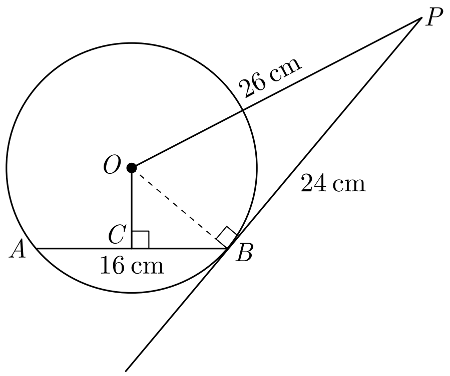
Thus $$OP = \sqrt{569} = 23.85 \text{ cm}$$

117. AB is a chord of circle with centre O. At B, a tangent PB is drawn such that its length is 24 cm. The distance of P from the centre is 26 cm. If the chord AB is 16 cm, find its distance from the centre.



Ans : [Board Term-2 Delhi 2014, 2012]

We redraw the given figure by joining O to B as shown below.



Here we have drawn perpendicular OC on chord AB. Thus Triangle ΔOCB is also right angled triangle,

We have $PB = 24$ cm, $OP = 26$ cm.

Triangle ΔOPB is right angled triangle because PB is tangent at radius OB and $\angle OPB = 90^\circ$.

In right angled ΔOPB, we have

$$OB = \sqrt{OP^2 - BP^2}$$

$$= \sqrt{26^2 - 24^2}$$

$$= \sqrt{676 - 576} = \sqrt{100}$$

$$= 10 \text{ cm}$$

Since perpendicular drawn from the centre to a chord bisect it, we have

$$BC = \frac{1}{2}AB = \frac{16}{2} = 8 \text{ cm}$$

Now in $\Delta\, OBC$, $OC^2 = OB^2 - BC^2$

$$= 10^2 - 8^2 = 36$$

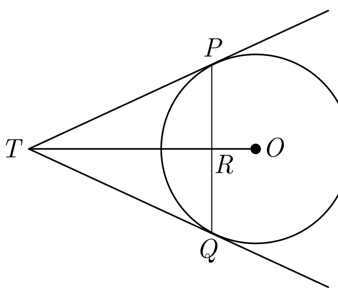
$$OC = 6 \text{ cm}$$

Thus distance of the chord from the centre is 6 cm.

118. From a point T outside a circle of centre O, tangents TP and TQ are drawn to the circle. Prove that OT is the right bisector of line segment PQ.

Ans : [Board Term-2 Delhi 2015]

A circle with centre O. Tangents TP and TQ are drawn from a point T outside a circle as shown in figure below.



Since length of tangents from an external point to a circle are equal,

$$TP = TQ$$

Angle $\angle TPR$ and $\angle TQR$ are opposite angle of equal sides, thus

$$\angle TPR = \angle TQR$$

Now in $\Delta\, PTR$ and $\Delta\, QTR$

$$TP = TQ$$

$$TR = TR \quad \text{(Common)}$$

$$\angle TPR = \angle TQR$$

Thus $\Delta\, PTR \cong \Delta\, QTR$

and $PR = QR$

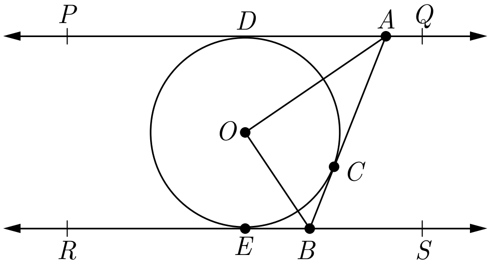
and $\angle PRT = \angle QRT$

But $\angle PRT + QRT = 180^\circ$ as PQ is line segment,

$$\angle PRT = \angle QRT = 90^\circ$$

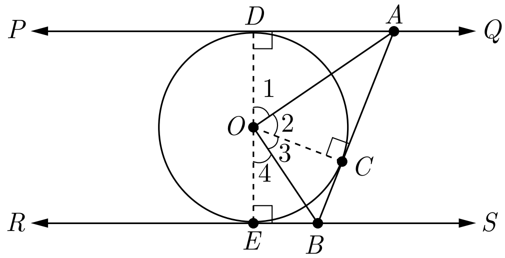
Therefore TR or OT is the right bisector of line segment PQ. Hence proved.

119. In Figure, PQ and RS are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting PQ at A and RS at B. Prove that $\angle AOB = 90^\circ$.



Ans : [Board 2019 OD STD, 2014, 2012]

We redraw the given figure as shown below.



In $\Delta\, DOA$ and $\Delta\, COA$, DA and AC are tangents drawn from common point,

Thus $DA = AC$

Due to angle between tangent and radius,

$$\angle ODA = \angle OCA = 90^\circ$$

Due to radius of circle,

$$OD = OC$$

By SAS symmetry we have

$$\Delta\, DOA \approx \Delta\, COA$$

Hence, by CPCT, $\angle 1 = \angle 2$

i.e., $\angle DOA = \angle COA$ ...(1)

Similarly, by SAS

$$\Delta\, BOC = \Delta\, BOE$$

and by CPCT $\angle 3 = \angle 4$

i.e., $\angle COB = \angle BOE$ ...(2)

Now, angles on a straight line,

$$\angle 1 + \angle 2 + \angle 3 + \angle 4 = 180^\circ$$

From equation (1) and (2) we have

$$2\angle 2 + 2\angle 3 = 180^\circ$$

$$\angle 2 + \angle 3 = 90^\circ$$

i.e., $\angle AOC + \angle BOC = 90°$

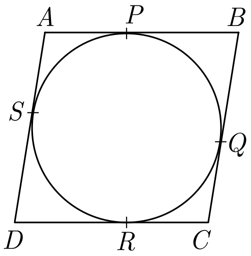
or $\angle AOB = 90°$ Hence Proved

120. Prove that the parallelogram circumscribing a circle is a rhombus.

Ans : [Board 2020 Delhi STD, 2013, 2014]

Let $ABCD$ be the parallelogram.

$$AB = CD, AD = BC \qquad (1)$$



Since length of tangents from an external point to a circle are equal,

At A, $AP = AS$ (2)

At B $BP = BQ$ (3)

At C $CR = CQ$ (4)

At D $DR = DS$ (5)

Adding above 4 equation we have

$$AP + PB + CR + DR = AS + BQ + CQ + DS$$

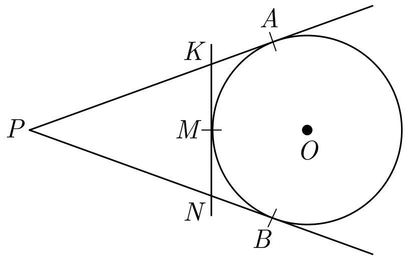
or, $AB + CD = AD + BC$

From (1) $2AB = 2AD$

or $AB = AD$

Thus $ABCD$ is a rhombus.

121. In given figure, PA and PB are tangents from a point P to the circle with centre O. At the point M, other tangent to the circle is drawn cutting PA and PB at K and N. Prove that the perimeter of $\Delta PNK = 2PB$.



Ans : [Board Term-2, 2012]

Since length of tangents from an external point to a circle are equal,

$$PA = PB$$

$$KM = KA$$

$$MN = BN$$

Now $KN = KM + MN$

$$= KA + BN$$

Now perimeter of ΔPNK

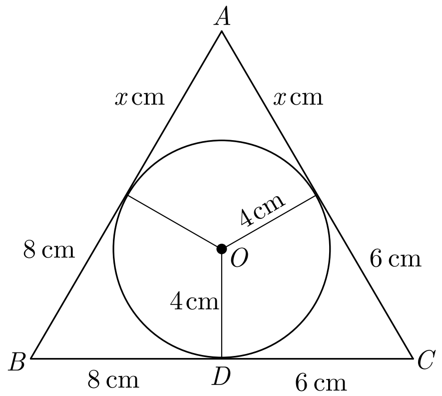
$$p = PN + KN + PK$$

$$= PN + BN + KA + PK$$

$$= PB + PA$$

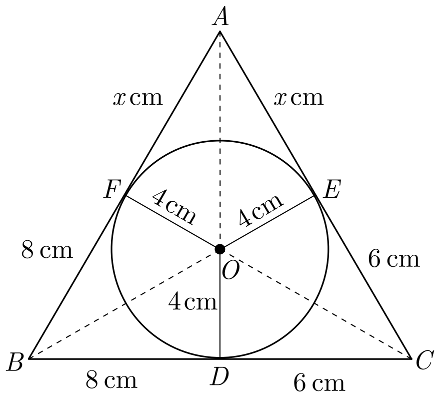
$$= 2PB \qquad (PA = PB)$$

122. In the figure, the ΔABC is drawn to circumscribe a circle of radius 4 cm, such that the segments BD and DC are of lengths 8 cm and 6 cm respectively. Find AB and AC.



Ans : [Board Term-2 Delhi 2014, 2012]

We redraw the given circle by joining AO, BO and CO shown in figure below. Let length of AF be x.



Since length of tangents from an external point to a circle are equal,

At A, $\quad AF = AE = x \quad (2)$

At B $\quad BF = BD = 8$ cm $\quad (3)$

At C $\quad CD = CE = 6$ cm $\quad (4)$

Now $\quad AB = x+8$

$$AC = x+6$$

$$BC = 8+6 = 14 \text{ cm}$$

Perimeter of circle

$$p = AB+BC+CA$$

$$= x+8+14+x+6$$

$$= 2(x+14)$$

Semi-perimeter of circle

$$s = \tfrac{1}{2}p = x+14$$

Area or triangle $\Delta\, ABC$

$$\Delta\, ABC = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{48x^2+672x} \quad (1)$$

Area or triangle $\Delta\, ABC$,

$$\Delta\, ABC = \tfrac{1}{2}rp$$

$$= \tfrac{1}{2}\times 4\times 2(x+14)$$

$$= 4(x+14) \quad (2)$$

From equation (1) and (2) we have

$$48x^2+672x = 16(x+14)^2$$

$$48x(x+14) = 16(x+14)^2$$

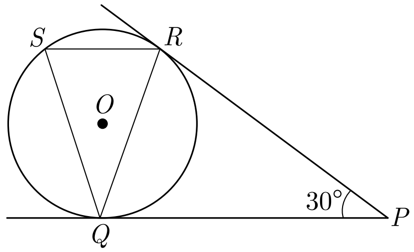
$$3x = x+14$$

or, $\quad x = 7$

Thus $\quad AC = 6+7 = 13$ cm

and $\quad AB\ 8+7 = 15$ cm.

123. In the figure, tangents PQ and PR are drawn from an external point P to a circle with centre O, such that $\angle RPQ = 30°$. A chord RS is drawn parallel to the tangent PQ. Find $\angle RQS$.



Ans : [Board Term-2 Delhi 2015]

Since length of tangents from an external point to a circle are equal,

$$PR = PQ$$

Now $\quad \angle PRQ = \angle PQR = \dfrac{180° - 30°}{2}$

$$= \frac{150°}{2} = 75°$$

Since $SR \parallel QP$, $\angle SRQ$ and $\angle RQP$ are alternate angle,

$$\angle SRQ = \angle RQP = 75°$$

Thus $\quad SQ = RQ$

and $\quad \angle RSQ = \angle SRQ = 75°$

In triangle $\Delta\, AQR$,

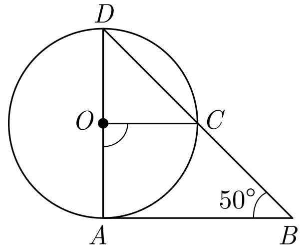
$$\angle SQR + \angle QSR + \angle QRS = 180°$$

$$\angle SQR + 75° + 75° = 180°$$

$$\angle SQR = 180° - 150° = 30°$$

Thus $\angle SQR = 30°$.

124. In the given figure, AD is a diameter of a circle with centre O and AB is a tangent at A. C is a point on the circle such that DC produced intersects the tangent at B and $\angle ABC = 50^\circ$. Find $\angle AOC$.



Ans : [Board Term-2 2015]

Tangent drawn at any point of a circle is perpendicular to the radius through the point contact.

Therefore $\angle A = 90^\circ$

Now in ΔDAB we have

$$\angle D + \angle A + \angle B = 180^\circ$$

$$\angle D + 90^\circ + 50^\circ = 180^\circ$$

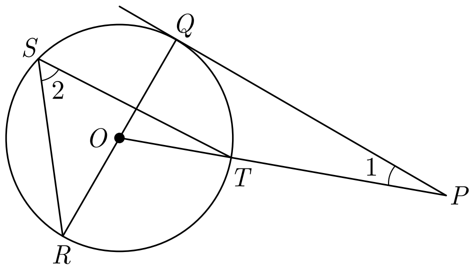
$$\angle D = 40^\circ$$

Angle subtended at the centre is always 2 time of angle subtended at circumference by same arc. Thus

$$\angle AOC = 2\angle ADC = 2\angle D$$

$$= 2 \times 40^\circ = 80^\circ$$

125. In figure PQ is a tangent from an external point P to a circle with centre O and OP cuts the circle at T and $\angle QOR$ is a diameter. If $\angle POR = 130^\circ$ and S is a point on the circle, find $\angle 1 + \angle 2$.



Here $\angle OQP = 90^\circ$ because radius is always perpendicular to tangent at point of contact.

Angle subtended at the centre is always 2 time of angle subtended at circumference by same arc. Thus

$$\angle 2 = \frac{1}{2}\angle TOR = \frac{1}{2}\angle POR$$

$$= \frac{1}{2} \times 130^\circ \quad 65^\circ$$

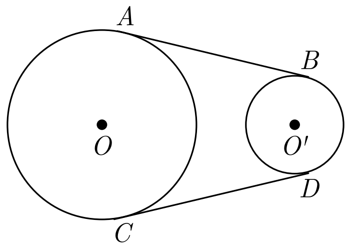
Now $\angle POQ = 180^\circ - 130^\circ = 50^\circ$

$$\angle 1 = 180^\circ - \angle OQP - \angle POQ$$

$$= 180^\circ - 90^\circ - 50^\circ = 40^\circ$$

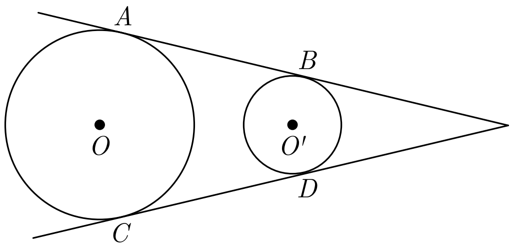
Now $\angle 2 + \angle 1 = 65^\circ + 40^\circ = 105^\circ$

126. In the figure AB and CD are common tangents to two circles of unequal radii. Prove that $AB = CD$.



Ans : [Board Term-2 Delhi Compt. 2017]

We redraw the given figure by extending AB and BD which intersect at P as shown in figure below



Since length of tangents from an external point to a circle are equal,

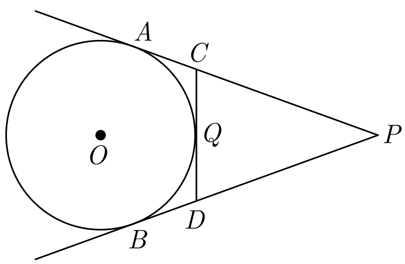
$$PA = PC$$

and $PB = PD$

Now, $PA - PB = PC - PD$

$AB = CD$ Hence Proved

127. In the given figure, PA and PB are tangents to the circle from an external point P. CD is another tangent touching the circle at Q. If $PA = 12$ cm, $QC = QD = 3$ cm, then find $PC + PD$.



Ans : [Board Term-2 Delhi Compt. 2017]

Since length of tangents from an external point to a circle are equal,

$$CA = CQ = 3 \text{ cm}$$

$$DQ = DB = 3 \text{ cm}$$

and $$PB = PA = 12 \text{ cm}$$

$$PA + PB = PC + CA + PD + DB$$

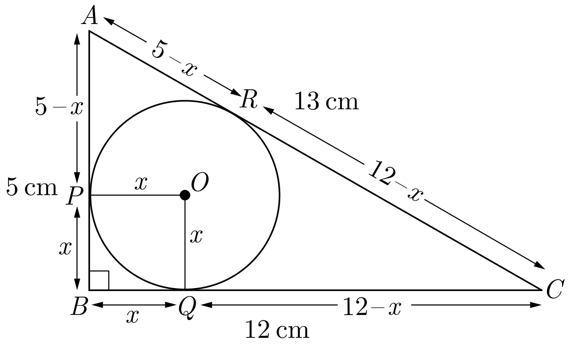
$$PC + PD = PA - CA + PB - DB$$

$$= 12 - 3 + 12 - 3 = 18 \text{ cm}$$

128. In a right angle $\Delta ABC, BC = 12$ cm and $AB = 5$ cm. Find the radius of the circle inscribed in this triangle.

Ans : [Board Term-2 Delhi 2014]

Let the radius of circle be x. As per given in question we draw the figure shown below.



Since length of tangents from an external point to a circle are equal,

At A, $$AP = AR = 5 - x \quad (1)$$

At B $$BP = BQ = x \quad (2)$$

At C $$CR = CQ = 12 - x \quad (3)$$

Here, $AB = 5$ cm, $BC = 12$ cm and $\Delta B = 90^{\circ}$

Now $$AC = \sqrt{12^2 + 5^2} = \sqrt{144 + 25}$$

$$= \sqrt{169} = 13 \text{ cm}$$

Now $$AC = AR + RC$$

$$13 = 5 - x + 12 - x$$

$$2x = 17 - 13 = 4$$

$$x = \frac{4}{2} = 2 \text{ cm}$$

Hence, radius of the circle is 2 cm.