CHAPTER 9

SOME APPLICATIONS OF TRIGONOMETRY

ONE MARK QUESTIONS

Multiple Choice Questions

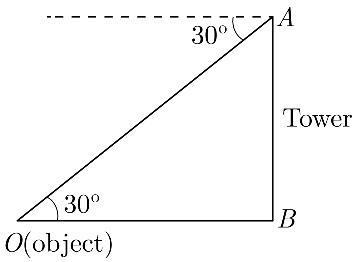
1. If the angle of depression of an object from a 75 m high tower is 30°, then the distance of the object from the tower is

(a) $25\sqrt{3}$ m (b) $50\sqrt{3}$ m

(c) $75\sqrt{3}$ m (d) 150 m

Ans :

We have
$$\tan 30^\circ = \frac{AB}{OB}$$
$$\frac{1}{\sqrt{3}} = \frac{75}{OB}$$
$$OB = 75\sqrt{3} \text{ m}$$



Thus (c) is correct option.

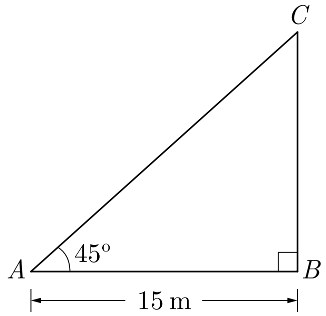
2. A tree casts a shadow 15 m long on the level of ground, when the angle of elevation of the sun is 45°. The height of a tree is

(a) 10 m (b) 14 m

(c) 8 m (d) 15 m

Ans : (d) 15 m

Let BC be the tree of height h meter. Let AB be the shadow of tree.



In ΔABC,
$$CB = 90^\circ$$
$$\frac{BC}{BA} = \tan 45^\circ$$
$$BC = AB = 15 \text{ m}$$

Thus (d) is correct option.

3. If the height and length of the shadow of a man are equal, then the angle of elevation of the sun is,

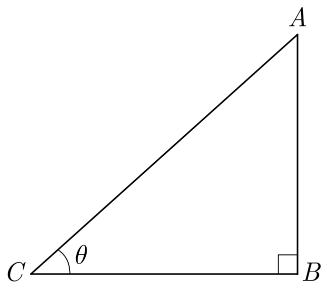
(a) 45° (b) 60°

(c) 90° (d) 120°

Ans :

Let AB be the height of a man and BC be the shadow of a man.
$$AB = BC$$

In ΔABC,
$$\tan\theta = \frac{AB}{BC}$$
$$\frac{AB}{AB} = \tan\theta$$
$$\tan\theta = 1 \Rightarrow \theta = 45^\circ$$



Thus (a) is correct option.

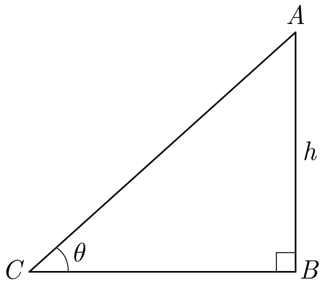
4. The ratio of the length of a rod and its shadow is $1:\sqrt{3}$ then the angle of elevation of the sun is

(a) 90° (b) 45°

(c) 30° (d) 75°

Ans :

Let AB be the rod of length h, BC be its shadow of length $\sqrt{3}\,h$, θ be the angle of elevation of the sun.

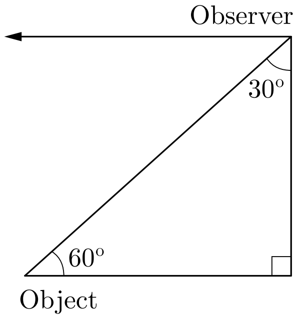


In ΔABC, $\dfrac{h}{\sqrt{3}\,h} = \tan\theta$

$$\tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30^\circ$$

Thus (c) is correct option.

5. In the given figure, the positions of the observer and the object are mentioned, the angle of depression is



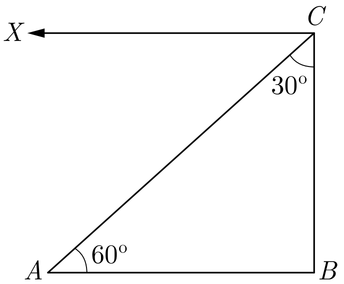
(a) 30° (b) 90°

(c) 60° (d) 45°

Ans :

$$\angle XCA = \angle CAB = 60^\circ$$

Hence, angle of depression $= 60^\circ$



Thus (c) is correct option.

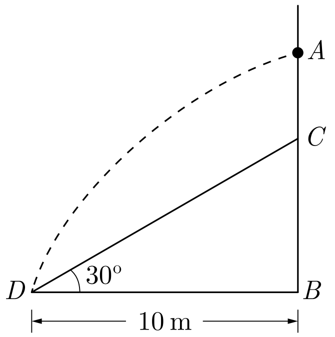
6. A tree is broken by the wind. The top struck the ground at an angle of 30° and at distance of 10 m from its root. The whole height of the tree is $(\sqrt{3} = 1.732)$

(a) $10\sqrt{3}$ m (b) $3\sqrt{10}$ m

(c) $20\sqrt{3}$ m (d) $3\sqrt{20}$ m

Ans :

Let AB be the tree of height x, and AC be the broken part of tree.



Now $AC = CD$

$\angle CDB = 30^\circ$

$BD = 10\text{ m}$

In ΔCDB, $\tan 30^\circ = \dfrac{CB}{DB} = \dfrac{CB}{10}$

$$\frac{1}{\sqrt{3}} = \frac{CB}{10}$$

$$CB = \frac{10}{\sqrt{3}}$$

Also, $\cos 30^\circ = \dfrac{DB}{DC} = \dfrac{10}{DC}$

$$DC = \frac{20}{\sqrt{3}} = AC$$

Height of tree,

$$AC + CB = \frac{20}{\sqrt{3}} + \frac{10}{\sqrt{3}} = \frac{30}{\sqrt{3}}$$

$$= 10\sqrt{3} \text{ m}$$

Thus (a) is correct option.

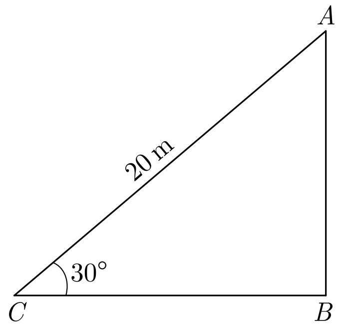
7. A circle artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground, then the height of pole, if the angle made by the rope with the ground level is 30°, is

(a) 5 m (b) 10 m

(c) 15 m (d) 20 m

Ans :

Let AB be the vertical pole and CA be the 20 m long rope such that its one end A is tied from the top of the vertical pole AB and the other end C is tied to a point C on the ground.



In $\triangle ABC$, we have

$$\sin 30° = \frac{AB}{AC}$$

$$\frac{1}{2} = \frac{AB}{AC}$$

$$\frac{1}{2} = \frac{AB}{20} \Rightarrow AB = 10 \text{ m}$$

Hence, the height of the pole is 10 m.

Thus (b) is correct option.

8. The length of a string between a kite and a point on the ground is 85 m. If the string makes an angle θ with level ground such that $\tan\theta = \frac{15}{8}$, then the height of kite is

(a) 75 m (b) 78.05 m

(c) 226 m (d) None of these

Ans :

Length of the string of the kite,

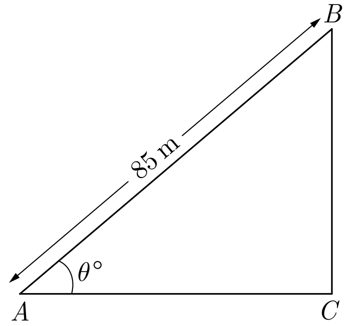
$$AB = 85 \text{ m}$$

and $$\tan\theta = \frac{15}{8}$$

$$\cot\theta = \frac{8}{15}$$

$$\text{cosec}^2\theta - 1 = \frac{64}{225}$$

$$\text{cosec}^2\theta = 1 + \frac{64}{225} = \frac{289}{225}$$



$$\text{cosec}\,\theta = \sqrt{\frac{289}{225}} = \frac{17}{15}$$

$$\sin\theta = \frac{15}{17}$$

In $\triangle ABC$, $$\sin\theta = \frac{BC}{AB}$$

$$\frac{15}{17} = \frac{BC}{85} \Rightarrow BC = 75 \text{ m}$$

Thus height of kite is 75 m.

Thus (a) is correct option.

9. The top of two poles of height 20 m and 14 m are connected by a wire. If the wire makes an angle of 30° with the horizontal, then the length of the wire is

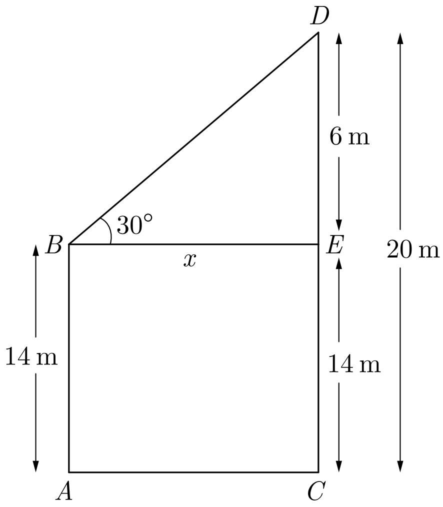
(a) 12 m (b) 10 m

(c) 8 m (d) 6 m

Ans :

Height of big pole $CD = 20 \text{ m}$

Height of small pole $AB = 14 \text{ m}$



$$DE = CD - CE$$
$$= CD - AB \qquad [AB = CE]$$
$$= 20 - 14 = 6 \text{ m}$$

In $\triangle BDE$, $\sin 30^\circ = \dfrac{DE}{BD}$

$$\frac{1}{2} = \frac{6}{BD} \Rightarrow BD = 12 \text{ m}$$

Thus length of wire is 12 m.

Thus (a) is correct option.

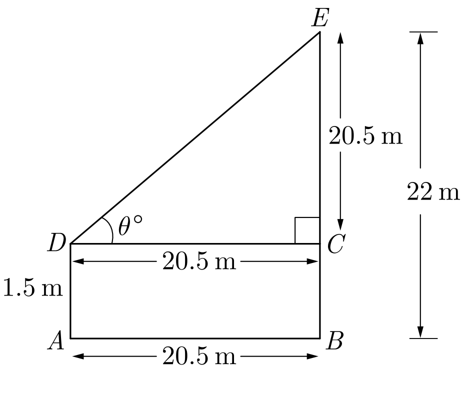
10. An observer, 1.5 m tall is 20.5 away from a tower 22 m high, then the angle of elevation of the top of the tower from the eye of observer is

(a) 30° (b) 45°

(c) 60° (d) 90°

Ans :

Let $BE = 22$ m be the height of the tower and $AD = 1.5$ m be the height of the observer. The point D be the observer's eye. We draw $DC \parallel AB$ as shown below.



Then, $AB = 20.5 \text{ m} = DC$

and $EC = BE - BC = BE - AD$

$$= 22 - 1.5 = 20.5 \text{ m} \qquad [BC = AD]$$

Let θ be the angle of elevation make by observer's eye to the top of the tower i.e. $\angle DCE$,

$$\tan\theta = \frac{P}{B} = \frac{CE}{DC} = \frac{20.5}{20.5}$$
$$\tan\theta = 1$$
$$\tan\theta = \tan 45^\circ \Rightarrow \theta = 45^\circ$$

Thus (b) is correct option.

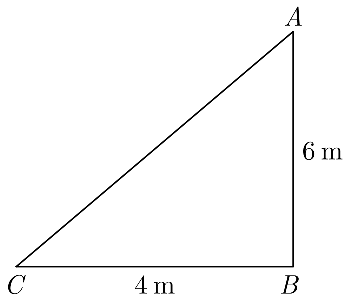
11. A 6 m high tree cast a 4 m long shadow. At the same time, a flag pole cast a shadow 50 m long. How long is the flag pole?

(a) 75 m (b) 100 m

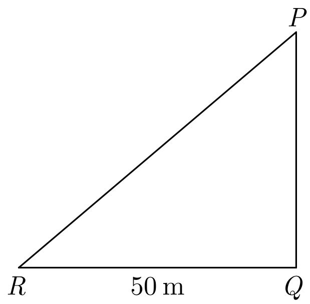
(c) 150 m (d) 50 m

Ans : (a) 75 m

Let AB be height of tree and BC its shadow.



Again, let PQ be height of pole and QR be its shadow. At the same time, the angle of elevation of tree and poles are equal i.e $\triangle ABC \sim PQR$



Thus $\dfrac{AB}{BC} = \dfrac{PQ}{QR}$

$$\frac{6}{4} = \frac{PQ}{50}$$

$$PQ = \frac{50 \times 6}{4} = 75 \text{ m}$$

Thus (a) is correct option.

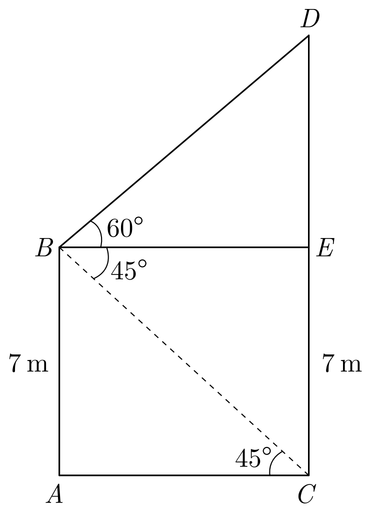
12. From the top of a 7 m high building the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°, then the height of the tower is

(a) 14.124 m (b) 17.124 m

(c) 19.124 m (d) 15.124 m

Ans :

Let AB be the building and CD be the tower. We draw $BE \perp CD$ as shown below.



Here $CE = AB = 7$ m

$\angle EBD = 60°$

and $\angle ACB = \angle CBE = 45°$

From $\triangle ACB$, we have

$$\cot 45° = \frac{AC}{AB}$$

$$\frac{AC}{7} = 1 \Rightarrow AC = 7 \text{ m}$$

$$BE = AC = 7 \text{ m}$$

From $\triangle EBD$, we have

$$\tan 60° = \frac{DE}{BE}$$

$$\frac{DE}{7} = \sqrt{3} \Rightarrow DE = 7\sqrt{3} \text{ m}$$

$$\text{Height of the tower} = (7 + 7\sqrt{3}) = 7(\sqrt{3} + 1)$$

$$= 7(1.732 + 1) = 7 \times 2.732$$

$$= 19.124 \text{ m}$$

Thus (c) is correct option.

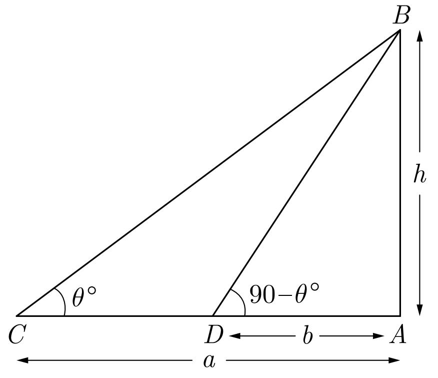
13. The angles of elevation of the top of a tower from the points P and Q at distance of a and b respectively from the base and in the same straight line with it, are complementary. The height of the tower is

(a) ab (b) $\sqrt{ab}$

(c) $\sqrt{\frac{a}{b}}$ (d) $\sqrt{\frac{b}{a}}$

Ans :

Let AB be the tower. Let C and D be two points at distance a and b respectively from the base of the tower.



In $\triangle ABC$, $\tan\theta = \frac{AB}{AC}$

$$\tan\theta = \frac{h}{a} \quad ...(1)$$

In $\triangle ABD$, $\tan(90° - \theta) = \frac{AB}{AD}$

$$\cot\theta = \frac{h}{b} \quad ...(2)$$

From equation (1) and (2), we have

$$\tan\theta \times \cot\theta = \frac{h}{a} \times \frac{h}{b}$$

$$1 = \frac{h^2}{ab} \Rightarrow h = \sqrt{ab}$$

Thus (b) is correct option.

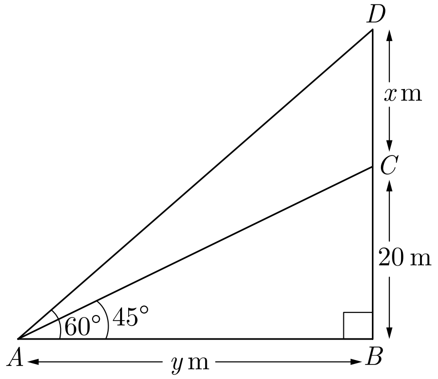
14. From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively, then the height of the tower is

(a) 14.64 m (b) 28.64 m

(c) 38.64 m (d) 19.64 m

Ans :

Let the height of the building be BC, $BC = 20$ m and height of the tower be CD Let the point A be at a distance y from the foot of the building.



Now, in $\triangle ABC$, $\frac{BC}{AB} = \tan 45^\circ = 1$

$$\frac{20}{y} = 1 \Rightarrow y = 20 \text{ m}$$

i.e. $AB = 20$ m

Now, in $\triangle ABC$, $\frac{BD}{AB} = \tan 60^\circ = \sqrt{3}$

$$\frac{BD}{AB} = \sqrt{3}$$

$$\frac{20+x}{20} = \sqrt{3}$$

$$20 + x = 20\sqrt{3}$$

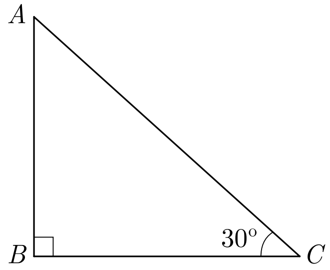
$$x = 20\sqrt{3} - 20$$

$$= 20 \times 0.732$$

$$= 14.64 \text{ m}$$

Thus (a) is correct option.

15. Assertion : In the figure, if $BC = 20$ m, then height AB is 11.56 m.



Reason : $\tan\theta = \frac{AB}{BC} = \frac{\text{perpendicular}}{\text{base}}$ where θ is the angle $\angle ACB$.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

We have $\tan 30^\circ = \frac{AB}{BC} = \frac{AB}{20}$

$$AB = \frac{1}{\sqrt{3}} \times 20 = \frac{20}{1.73} = 11.56 \text{ m}$$

Both the assertion and reason are correct, reason is the correct explanation of the assertion.

Thus (a) is correct option.

Fill in the Blank Questions

16. The .... of an object viewed, is the angle formed by the line of sight with the horizontal when it is above the horizontal level, i.e., the case when we raise our head to look at the object.

Ans :

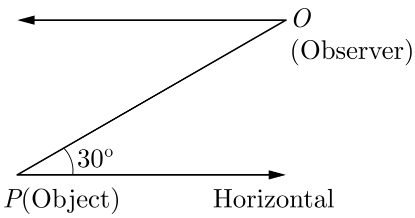
angle of elevation

17. The .... of an object viewed, is the angle formed by the line of sight with the horizontal when it is below the horizontal level, i.e., the case when we lower our head to look at the object.

Ans :

angle of depression

18. In the adjoining figure, the positions of observer and object are marked. The angle of depression is ....



Ans :

30°

19. The .... is the line drawn from the eye of an observer to the point in the object viewed by the observer.

Ans :

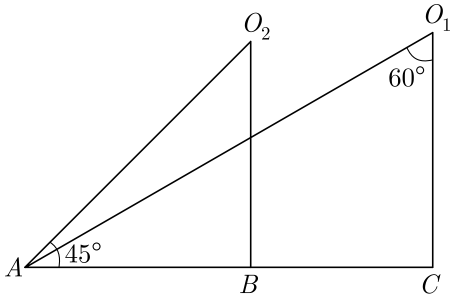
line of sight

20. .... are used to find height or length of an object or distance between two distant objects.

Ans :

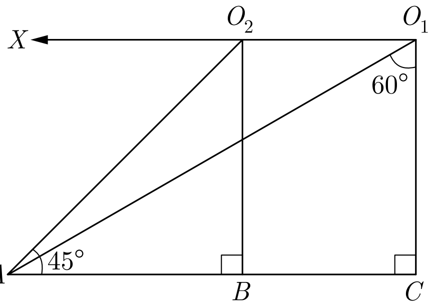
Trigonometric ratios

21. In Figure, the angles of depressions from the observing positions O_1 and O_2 respectively of the object A are ....



Ans : [Board 2020 OD Standard]

Here we have drawn O_1X parallel to AC.



$$\angle AO_1X = 90^\circ - 60^\circ = 30^\circ$$

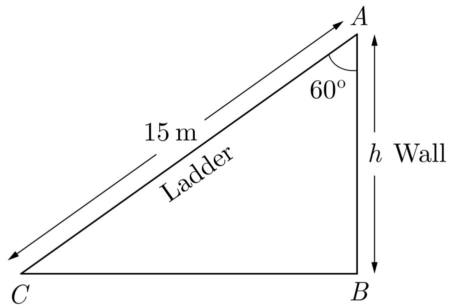
$$\angle AO_2X = \angle BAO_2 = 45^\circ$$

VERY SHORT ANSWER QUESTIONS

22. A ladder 15 m long leans against a wall making an angle of 60^o with the wall. Find the height of the point where the ladder touches the wall.

Ans : [Board Term-2 2014]

Let the height of wall be h. As per given in ques we have drawn figure below.



$$\frac{h}{15} = \cos 60^o$$

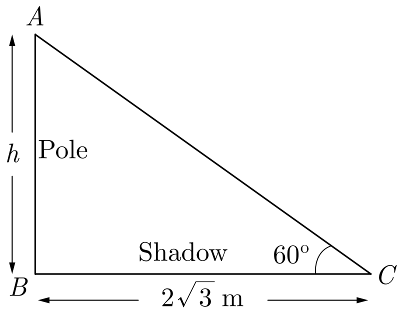
$$h = 15 \times \cos 60^o$$

$$= 15 \times \frac{1}{2} = 7.5 \text{ m}$$

23. A pole casts a shadow of length $2\sqrt{3}$ m on the ground, when the Sun's elevation is 60^o. Find the height of the pole.

Ans : [Board Term-2 Foreign 2015]

Let the height of pole be h. As per given in question we have drawn figure below.



Now $$\frac{h}{2\sqrt{3}} = \tan 60^o$$

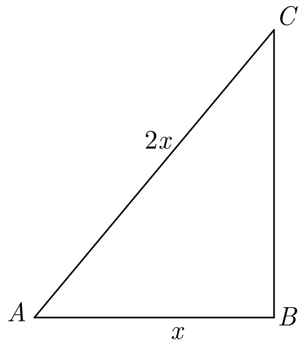
$$h = 2\sqrt{3}\tan 60^o$$

$$= 2\sqrt{3} \times \sqrt{3} = 6 \text{ m}$$

24. If the length of the ladder placed against a wall is twice the distance between the foot of the ladder and the wall. Find the angle made by the ladder with the horizontal.

Ans : [Board Term-2 2015]

Let the distance between the foot of the ladder and the wall is x, then length of the ladder will be $2x$. As per given in question we have drawn figure below.



In ΔABC, $\angle B = 90^\circ$

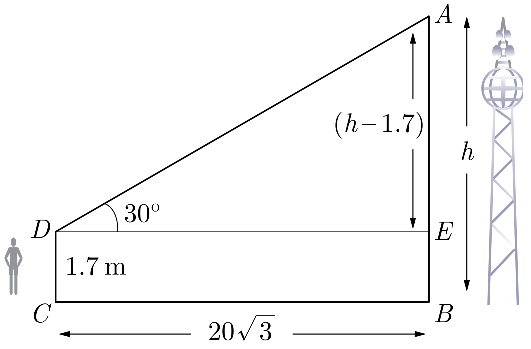
$$\cos A = \frac{x}{2x} = \frac{1}{2} = \cos 60^\circ$$

$$A = 60^\circ$$

25. An observer, 1.7 m tall, is $20\sqrt{3}$ m away from a tower. The angle of elevation from the eye of observer to the top of tower is 30°. Find the height of tower.

Ans : [Board Term-2 Foreign 2016]

Let height of the tower AB be h. As per given in question we have drawn figure below.



Here $AE = h - 1.7$

and $BC = DE = 20\sqrt{3}$

In ΔADE, $\angle E = 90^\circ$

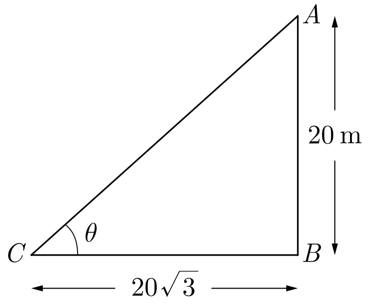
$$\tan 30^\circ = \frac{h-1.7}{20\sqrt{3}}$$

$$\frac{1}{\sqrt{3}} = \frac{h-1.7}{20\sqrt{3}}$$

$$h - 1.7 = 20$$

or $h = 20 + 1.7 = 21.7$ m

26. In figure, a tower AB is 20 m high and BC, its shadow on the ground, is $20\sqrt{3}$ m long. find the Sun's altitude.



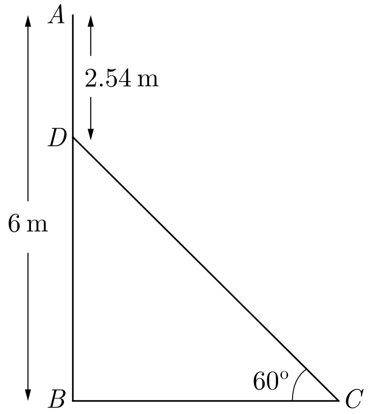
Ans : [Board Term-2 OD 2015]

Let the $\angle ACB$ be θ.

$$\tan\theta = \frac{AB}{BC} = \frac{20}{20\sqrt{3}} = \frac{1}{\sqrt{3}} = \tan 30^\circ$$

Thus $\theta = 30^\circ$

27. In the given figure, AB is a 6 m high pole and DC is a ladder inclined at an angle of 60° to the horizontal and reaches up to point D of pole. If $AD = 2.54$ m, find the length of ladder. (use $\sqrt{3} = 1.73$)



Ans : [Board Term-2 Delhi 2016]

We have $AD = 2.54$ m

$DB = 6 - 2.54 = 3.46$ m

In ΔBCD, $\angle B = 90^\circ$

$$\sin 60^\circ = \frac{BD}{DC}$$

$$\frac{\sqrt{3}}{2} = \frac{3.46}{DC}$$

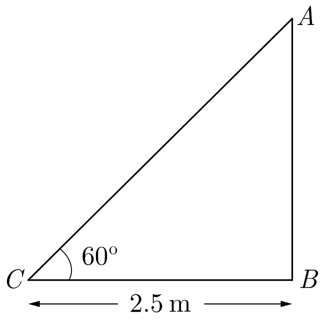
$$DC = \frac{3.46 \times 2}{\sqrt{3}} = \frac{3.46}{1.73} = 4$$

Thus length of ladder is 4 m.

28. A ladder, leaning against a wall, makes an angle of 60° with the horizontal. If the foot of the ladder is 2.5 m away from the wall, find the length of the ladder.

Ans : [Board Term-2 2011]

As per given in question we have drawn figure below.



In ΔACB with $\angle C = 60^\circ$, we get

$$\cos 60^\circ = \frac{2.5}{AC}$$

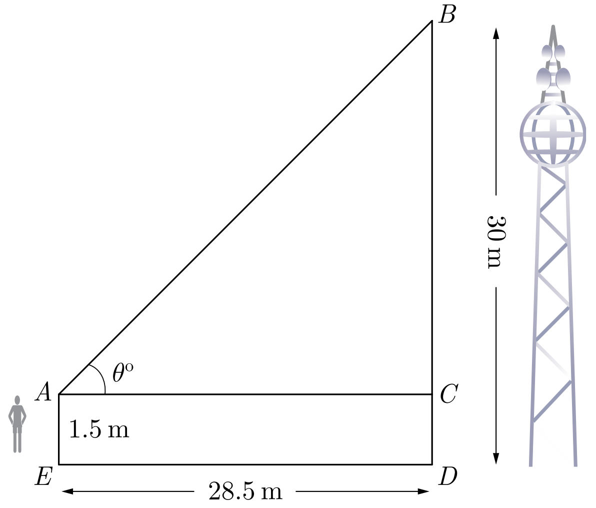
$$\frac{1}{2} = \frac{2.5}{AC}$$

$$AC = 2 \times 2.5 = 5 \text{ m}$$

29. An observer 1.5 m tall is 28.5 m away from a tower 30 m high. Find the angle of elevation of the top of the tower from his eye.

Ans : [Board Term-2 2012]

As per given in question we have drawn figure below.



Here $AE = 1.5$ m is height of observer and $BD = 30$ m m is tower.

Now $BC = 30 - 1.5 = 28.5$ m

In ΔBAC, $\tan\theta = \frac{BC}{AC}$

$$\tan\theta = \frac{28.5}{28.5} = 1 = \tan 45^\circ$$

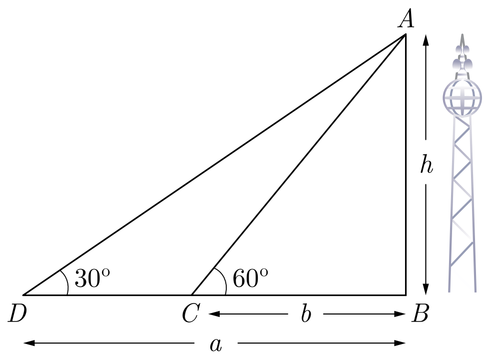
$$\theta = 45^\circ$$

Hence angle of elevation is 45°.

30. If the angles of elevation of the top of a tower from two points distant a and $b (a > b)$ from its foot and in the same straight line from it are respectively 30° and 60°, then find the height of the tower.

Ans : [Board Term-2 2014]

Let the height of tower be h. As per given in question we have drawn figure below.



From ΔABD, $\frac{h}{a} = \tan 30^\circ$

$$h = a \times \frac{1}{\sqrt{3}} = \frac{a}{\sqrt{3}} \quad \text{...(1)}$$

From ΔABC, $\frac{h}{b} = \tan 60^\circ$

$$h = b \times \sqrt{3} = b\sqrt{3} \quad \text{...(2)}$$

From (1) we get $a = \sqrt{3}\,h$

From (2) get $b = \frac{h}{\sqrt{3}}$

Thus $a \times b = \sqrt{3}\,h \times \frac{h}{\sqrt{3}}$

$$ab = h^2$$

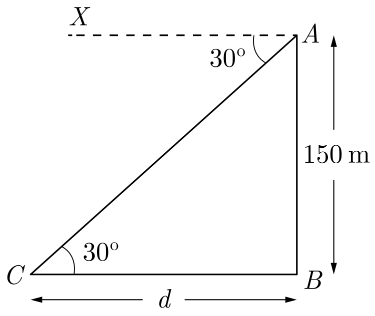
$$h = \sqrt{ab}$$

Hence, the height of the tower is $\sqrt{ab}$.

31. The angle of depression of a car parked on the road from the top of a 150 m high tower is 30°. Find the distance of the car from the tower (in m).

Ans : [Board Term-2, 2014]

Let the distance of the car from the tower be d. As per given in question we have drawn figure below.



Due to alternate angles we have

$$\angle CAX = \angle ACB = 30^\circ$$

In ΔABC, $\angle B = 90^\circ$

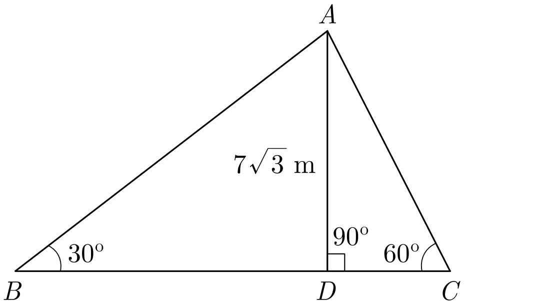
i110

$$\tan 30^\circ = \frac{150}{d}$$

$$\frac{1}{\sqrt{3}} = \frac{150}{d}$$

Thus $d = 150\sqrt{3}$ m.

32. In the given figure, if $AD = 7\sqrt{3}$ m, then find the value of BC.



Ans : [Board Term-2 2012]

Let $BD = x$ and $DC = y$

From ΔADB we get

$$\tan 30^\circ = \frac{7\sqrt{3}}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{7\sqrt{3}}{x}$$

$$x = 7\sqrt{3} \times \sqrt{3} = 21 \text{ m}$$

From ΔADC,

$$\tan 60^\circ = \frac{7\sqrt{3}}{y}$$

$$\sqrt{3} = \frac{7\sqrt{3}}{y}$$

$$y = 7 \text{ m.}$$

Now $BC = BD + DC$

$$= 21 + 7 = 28 \text{ m.}$$

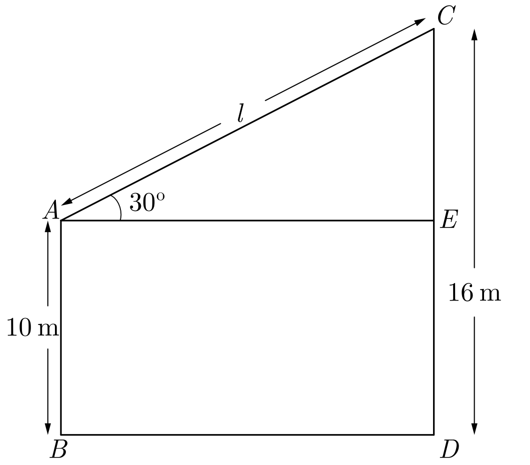
Hence, the value of BC is 28 m.

33. The top of two poles of height 16 m and 10 m are connected by a length l meter. If wire makes an angle of 30° with the horizontal, then find l.

Ans : [Board Term-2, 2012]

Let AB and CD be two poles, where $AB = 10$ m, $CD = 16$ m.

As per given in question we have drawn figure below.



Length $CE = CD - CE = CD - AB$

$$= 16 - 10 = 6 \text{ m.}$$

From ΔAEC, $\sin 30^\circ = \frac{CE}{l}$

$$\frac{1}{2} = \frac{CE}{l}$$

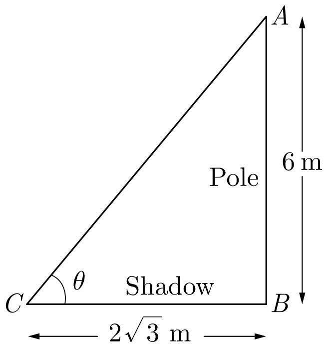
$$l = 2CE = 6 \times 2 = 12 \text{ m.}$$

Hence, the value of l is 12 m.

34. A pole 6 m high casts a shadow $2\sqrt{3}$ m long on the ground, then find the Sun's elevation.

Ans : [Board Term-2 2012]

Let the Sun's elevation be θ. As per given in question we have drawn figure below.



Length of pole is 6 m and length of shadow is $2\sqrt{3}$ m.

From ΔABC, we have

$$\tan\theta = \frac{AB}{BC} = \frac{6}{2\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3} = \tan 60^\circ$$

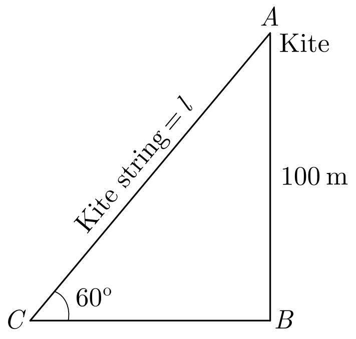
$$\theta = 60^\circ$$

Hence sun's elevation is 60°.

35. Find the length of kite string flying at 100 m above the ground with the elevation of 60°.

Ans : [Board Term-2, 2012]

Let the length of kite string $AC = l$. As per given in question we have drawn figure below.



Here $\angle ACB = 60^\circ$, height of kite $AB = 100$ m.

From ΔABC, we have

$$\sin 60^\circ = \frac{AB}{BC}$$

$$\frac{\sqrt{3}}{2} = \frac{100}{l}$$

$$l = \frac{2\times 100}{\sqrt{3}} = \frac{200}{\sqrt{3}} \text{ m}$$

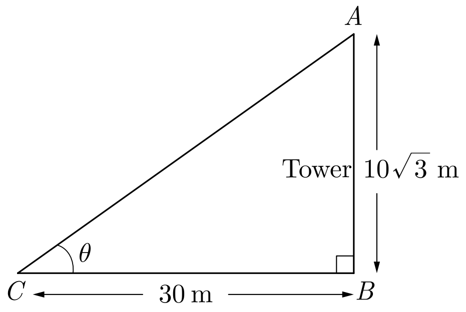
$$= \frac{200}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{200\sqrt{3}}{3} \text{ m}$$

Hence length the kite string is $\frac{200\sqrt{3}}{3}$

36. Find the angle of elevation of the top of the tower from the point on the ground which is 30 m away from the foot of the tower of height $10\sqrt{3}$ m.

Ans : [Board Term-2 2012]

Let the angle of elevation of top of the tower be θ. As per given in question we have drawn figure below.



From ΔABC,

$$\tan\theta = \frac{AB}{BC} = \frac{10\sqrt{3}}{30} = \frac{1}{\sqrt{3}} = \tan 30^\circ$$

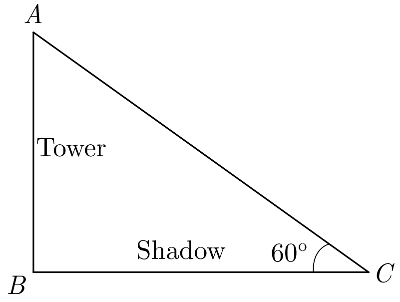
Thus $\theta = 30^\circ$

Hence angle of elevation is 30°.

37. If the altitude of the sun is 60°, what is the height of a tower which casts a shadow of length 30 m ?

Ans : [Board Term-2, 2011]

Let AB be the tower whose height be h. As per given in question we have drawn figure below.



Here shadow is $BC = 30$ m.

From ΔABC, we get

$$\frac{AB}{BC} = \tan 60^\circ$$

$$\frac{h}{30} = \sqrt{3}$$

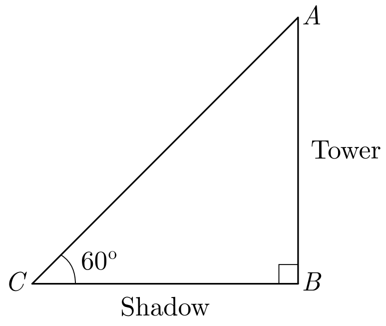
$$h = 30\sqrt{3} \text{ m}$$

Hence, height of tower is $30\sqrt{3}$ m.

38. The ratio of the height of a tower and the length of its shadow on the ground is $\sqrt{3} : 1$. What is the angle of elevation of the sun ?

Ans : [Board Term-2, 2016]

Let height of tower be AB and its shadow be BC. As per given in question we have drawn figure below.



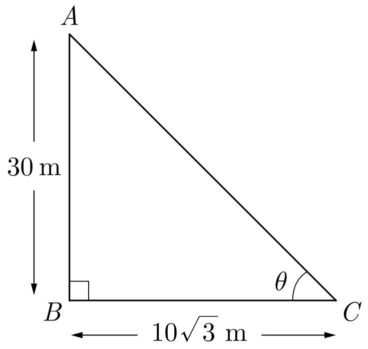
$$\frac{AB}{BC} = \tan\theta = \frac{\sqrt{3}}{1} = \tan 60^\circ$$

Hence, angle of elevation of sun is 60°.

39. If a tower 30 m high, casts a shadow $10\sqrt{3}$ m long on the ground, then what is the angle of elevation of the sun ?

Ans : [Board Term-2 OD 2017]

Tower AB is 30 m and shadow BC is $10\sqrt{3}$. As per given in question we have drawn figure below.



In right ΔABC we have,

$$\tan\theta = \frac{AB}{BC} = \frac{30}{10\sqrt{3}} = \sqrt{3} = \tan 60^\circ$$

Thus $\theta = 60^\circ$

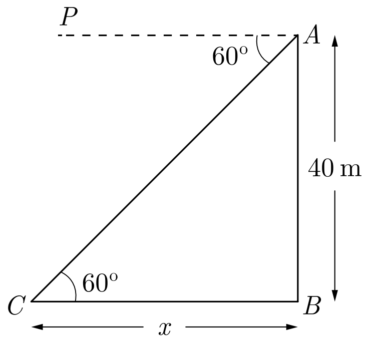
so, angle of elevation of sun is 60°.

TWO MARKS QUESTIONS

40. From the top of light house, 40 m above the water, the angle of depression of a small boat is 60°. Find how far the boat is from the base of the light house.

Ans : [Board Term-2 2015]

Let AB be the light house and C be the position of the boat. As per given in question we have drawn figure below.



Since $\angle PAC = 60^\circ \Rightarrow \angle ACB = 60^\circ$

Let $CB = x$. Now in ΔABC,

$$\tan 60^\circ = \frac{AB}{BC}$$

$$\sqrt{3} = \frac{40}{x}$$

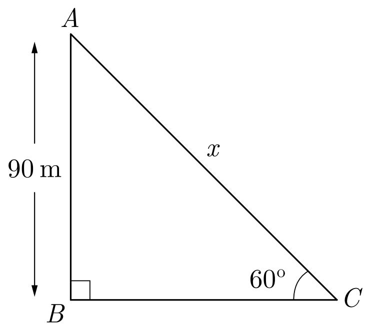
$$x = \frac{40}{\sqrt{3}} = \frac{40 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{40\sqrt{3}}{3} \text{ m}$$

Hence, the boat is $\frac{40\sqrt{3}}{3}$ m away from the foot of light house.

41. A kite is flying at a height of 90 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string assuming that there is no slack in the string.

Ans : [Board Term-2 2011, 2014]

As per given in question we have drawn figure below.



In right ΔABC, we have

$$\sin 60^\circ = \frac{AB}{AC}$$

$$\frac{\sqrt{3}}{2} = \frac{90}{x}$$

$$x = \frac{90 \times 2}{\sqrt{3}} = \frac{180}{\sqrt{3}} = \frac{3 \times 60}{\sqrt{3}}$$

$$= 60\sqrt{3}$$

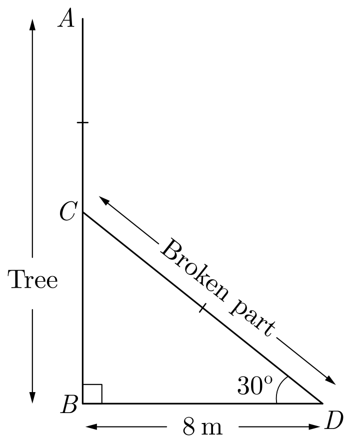
$$= 60 \times 1.732$$

Hence length of string is 103.92 m.

42. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.

Ans : [Board Term-2 2011]

Let the tree be AC and is broken at B. The broken part touches at the point D on the ground. As per given in question we have drawn figure below.



In right ΔCBD, $\cos 30^\circ = \frac{BD}{CD}$

$$\frac{\sqrt{3}}{2} = \frac{8}{CD}$$

$$CD = \frac{16}{\sqrt{3}}$$

and $\tan 30^\circ = \frac{BC}{BD}$

$$\frac{1}{\sqrt{3}} = \frac{BC}{8}$$

$$BC = \frac{8}{\sqrt{3}}$$

Height of tree,

$$BC + CD = \frac{8}{\sqrt{3}} + \frac{16}{\sqrt{3}}$$

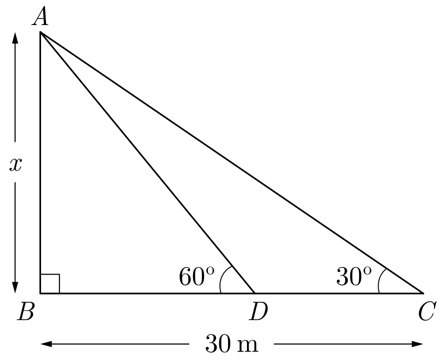
$$= \frac{24}{\sqrt{3}} = 8\sqrt{3}$$

Hence the height of the tree is $8\sqrt{3}$ m.

43. If the shadow of a tower is 30 m long, when the Sun's elevation is 30°. What is the length of the shadow, when Sun's elevation is 60° ?

Ans : [Board Term-2 2011]

As per given in question we have drawn figure below. Here AB is tower and BD is shadow at 60° and BC is shadow at 30° elevation.



In ΔABC, $\frac{AB}{BC} = \tan 30^\circ$

$$\frac{AB}{30} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$AB = \frac{30}{\sqrt{3}} = 10\sqrt{3}$$

In ΔABD, $\frac{AB}{BD} = \tan 60^\circ$

$$\frac{10\sqrt{3}}{BD} = \tan 60^\circ = \sqrt{3}$$

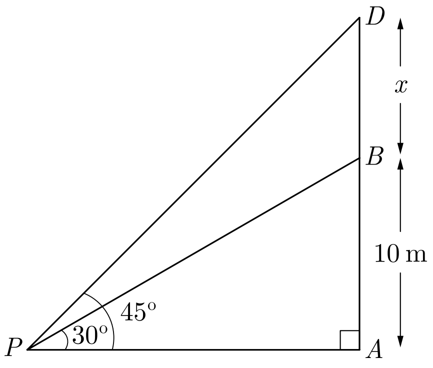
$$BD = 10 \text{ m}$$

Hence the length of shadow is 10 m.

44. From a point P on the ground the angle of elevation of the top of a 10 m tall building is 30°. A flag is hoisted at the top the of the building and the angle of elevation of the length of the flagstaff from P is 45°. Find the length of the flagstaff and distance of building from point P. [Take $\sqrt{3} = 1.732$]

Ans : [Board Term-2 2011, Delhi 2012, 2013]

Let height of flagstaff be $BD = x$. As per given in question we have drawn figure below.



$$\tan 30^\circ = \frac{AB}{AP}$$

$$\frac{1}{\sqrt{3}} = \frac{10}{AP}$$

$$AP = 10\sqrt{3}$$

Distance of the building from P,

$$= 10 \times 1.732 = 17.32 \text{ m}$$

Now $\tan 45^\circ = \frac{AD}{AP}$

$$1 = \frac{10+x}{17.32}$$

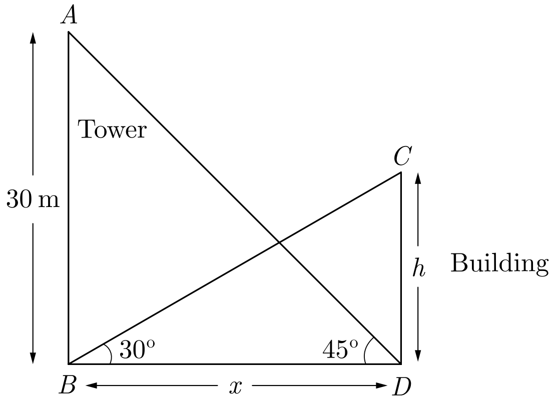
$$x = 17.32 - 10.00 = 7.32 \text{ m}$$

Hence, length of flagstaff is 7.32 m.

45. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 45°. If the tower is 30 m high, find the height of the building.

Ans : [Board Term-2 Delhi 2015]

Let the height of the building be $AB = h$. and distant between tower and building be, $BD = x$. As per given in question we have drawn figure below.



In ΔABD $\tan 45^\circ = \frac{AB}{BD}$

$$1 = \frac{30}{x}$$

$$x = 30 \quad \text{...(1)}$$

Now in ΔBDC,

$$\tan 30^\circ = \frac{CD}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{x}$$

$$\sqrt{3}\,h = x \Rightarrow h = \frac{x}{\sqrt{3}} \quad \text{...(2)}$$

From (1) and (2), we get

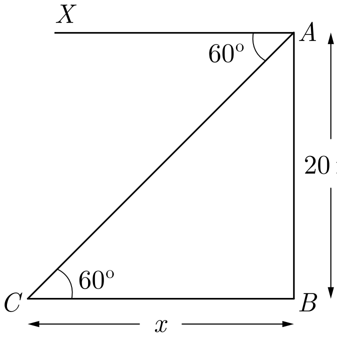
$$h = \frac{30}{\sqrt{3}} = 10\sqrt{3} \text{ m.}$$

Therefore height of the building is $10\sqrt{3}$ m

46. A player sitting on the top of a tower of height 20 m observes the angle of depression of a ball lying on the ground as 60°. Find the distance between the foot of the tower and the ball. Take $\sqrt{3} = 1.732$

Ans : [Board Term-2 2011]

Let C be the point where the ball is lying. As per given in question we have drawn figure below.



Due to alternate angles we obtain

$$\angle XAC = \angle ACB = 60^\circ$$

In ΔABC, $\tan 60^\circ = \frac{AB}{BC}$

$$\sqrt{3} = \frac{20}{x}$$

$$x = \frac{20}{\sqrt{3}} = 20\left(\frac{\sqrt{3}}{3}\right)$$

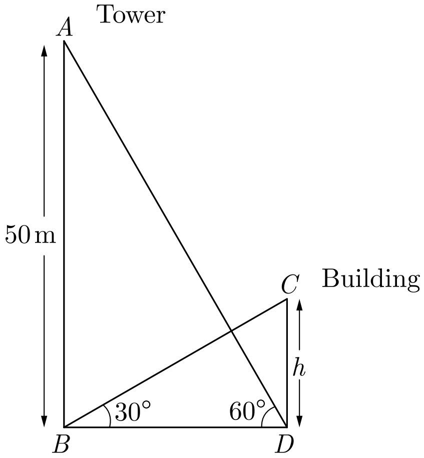
Hence, distance between ball and foot of tower is 11.53 m.

THREE MARKS QUESTIONS

47. The angle of elevation of the top of a building from the foot of a tower is 30° and the angle of elevation of the top of a tower from the foot of the building is 60°. If the tower is 50 m high, then find the height of the building.

Ans : [Board 2020 OD Standard]

As per given information in question we have drawn the figure below.



In ΔABD, $\tan 60^\circ = \frac{AB}{BD}$

$$\sqrt{3} = \frac{50}{BD}$$

$$BD = \frac{50}{\sqrt{3}}$$

Now in ΔBDC,

$$\tan 30^\circ = \frac{CD}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{\frac{50}{\sqrt{3}}} = \frac{h\sqrt{3}}{50}$$

$$3h = 50$$

$$h = \frac{50}{3} = 16.67$$

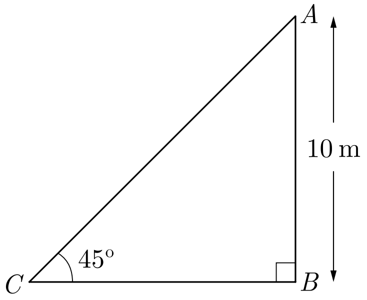
Hence, the height of the building is 16.67 m.

48. An electric pole is 10 m high. A steel wire tied to top of the pole is affixed at a point on the ground to keep the pole up right. If the wire makes an angle of 45° with the horizontal through the foot of the pole, find the length of the wire. [Use $\sqrt{2} = 1.414$]

Ans : [Board Term-2 2016]

Let OA be the electric pole and B be the point on the ground to fix the pole. Let BA be x.

As per given in question we have drawn figure below.



In ΔABC we have,

$$\sin 45^{\circ} = \frac{AB}{AC}$$

$$\frac{1}{\sqrt{2}} = \frac{10}{AC}$$

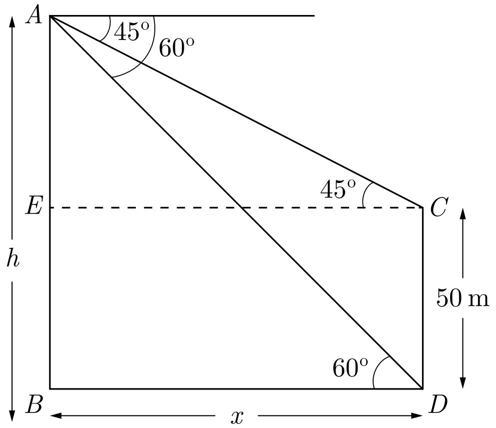
$$AC = 10\sqrt{2} = 10 \times 1.414 = 14.14 \text{ m}$$

Hence, the length of wire is 14.14 m

49. The angles of depression of the top and bottom of a 50 m high building from the top of a tower are 45° and 60° respectively. Find the height of the tower and the horizontal distance between the tower and the building. (Use $\sqrt{3} = 1.73$)

Ans : [Board Term-2 Delhi 2016]

As per given in question we have drawn figure below. Here AC is tower and DC is building.



We have $\tan 45^{\circ} = \frac{h-50}{x}$

$$x = h - 50 \quad ...(1)$$

and $\tan 60^{\circ} = \frac{h}{x}$

$$\sqrt{3} = \frac{h}{x}$$

$$x = \frac{h}{\sqrt{3}} \quad ...(2)$$

From (1) and (2) we have

$$h - 50 = \frac{h}{\sqrt{3}}$$

$$\sqrt{3}\,h - 50\sqrt{3} = h$$

$$\sqrt{3}\,h - h = 50\sqrt{3}$$

$$h(\sqrt{3} - 1) = 50\sqrt{3}$$

$$h = \frac{50\sqrt{3}}{\sqrt{3}-1} = \frac{50(3+\sqrt{3})}{2}$$

$$= 25\,(3+\sqrt{3})$$

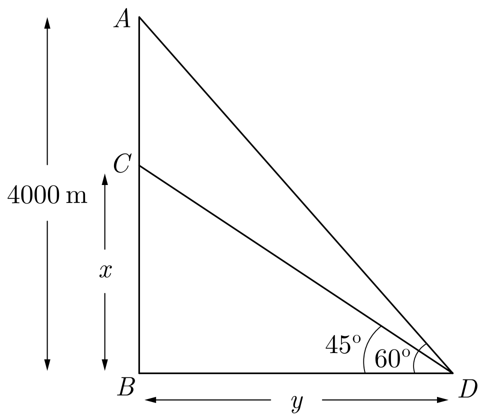
$$= 75 + 25\sqrt{3} = 118.25 \text{ m}$$

Thus $h = 118.25$ m.

50. An aeroplane, when flying at a height of 4000 m from the ground passes vertically above another aeroplane at an instant when the angles of elevation of the two planes from the same point on the ground are 60° and 45° respectively. Find the vertical distance between the aeroplanes at that instant. (Use $\sqrt{3} = 1.73$)

Ans : [Board Term-2 Foreign 2016]

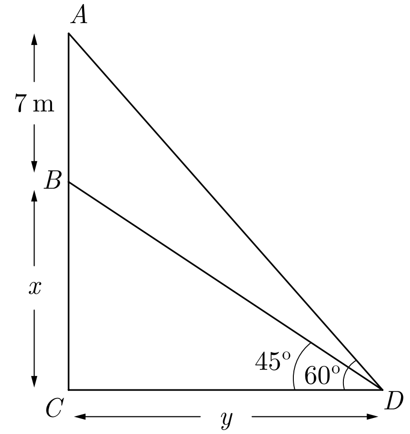
Let the height first plane be $AB = 4000$ m and the height of second plane be $BC = x$ m. As per given in question we have drawn figure below.



2306

$= 4000 \quad 2306$

1693

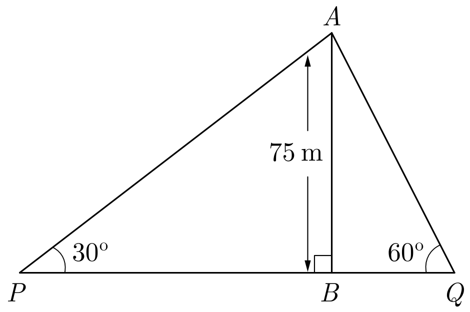


$.73) = 9.6$ m

52. Two men on either side of a 75 m high building and in line with base of building observe the angles of elevation of the top of the building as 30° and 60°. Find the distance between the two men. (Use $\sqrt{3} = 1.73$)

Ans : [Board Term-2 Foreign 2016]

Let AB be the building and the two men are at P and Q. As per given in question we have drawn figure below.



$$\text{In } \Delta ABP, \quad \tan 30^\circ = \frac{AB}{BP}$$

$$\frac{1}{\sqrt{3}} = \frac{75}{BP}$$

$$BP = 75\sqrt{3} \text{ m}$$

$$\text{In } \Delta ABQ, \quad \tan 60^\circ = \frac{AB}{BQ}$$

$$\sqrt{3} = \frac{75}{BQ}$$

$$BQ = \frac{75}{\sqrt{3}} = 25\sqrt{3}$$

Distance between the two men,

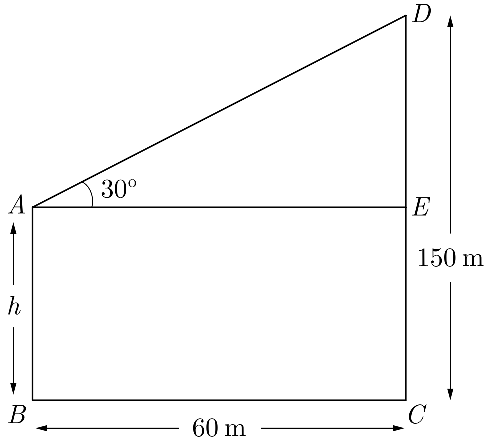
$$PQ = BP + BQ = 75\sqrt{3} + 25\sqrt{3}$$

$$= 100\sqrt{3} = 100 \times 1.73 = 173$$

53. The horizontal distance between two towers is 60 m. The angle of elevation of the top of the taller tower as seen from the top of the shorter one is 30°. If the height of the taller tower is 150 m, then find the height of the shorter tower.

Ans : [Board Term-2 2015]

Let AB and CD be two towers. Let the height of the shorter tower $AB = h$. As per given in question we have drawn figure below.



Here $BC = AE = 60$ m, $DE = DC - EC = (150 - h)$

In ΔAED, $\frac{DE}{AE} = \tan 30^o$

$$\frac{150 - h}{60} = \tan 30^o = \frac{1}{\sqrt{3}}$$

$$150\sqrt{3} - h\sqrt{3} = 60$$

$$\sqrt{3}\,h = 150\sqrt{3} - 60$$

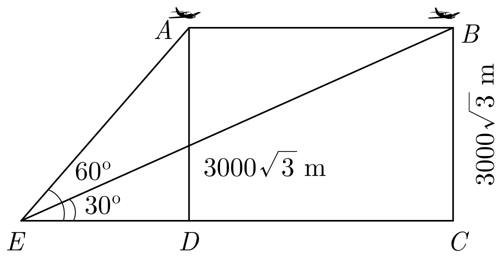
$$\sqrt{3}\,h = 150\sqrt{3} - 20\sqrt{3} \times \sqrt{3}$$

or $h = (150 - 20\sqrt{3})$ m

54. The angle of elevation of an aeroplane from a point on the ground is 60°. After a flight of 30 seconds the angle of elevation becomes 30°. If the aeroplane is flying at a constant height of $3000\sqrt{3}$ m, find the speed of the aeroplane.

Ans : [Board 2020 SQP Standard, 2014]

As per given in question we have drawn figure below. Here



$$\angle AED = 60^o, \angle BED = 30^o$$

$$AD = BC = 3000\sqrt{3} \text{ m}$$

Let the speed of the aeroplane be x.

$$AB = DC \times 30 \times x = 30x \text{ m} \quad ...(1)$$

In right ΔAED, we have

$$\tan 60^o = \frac{AD}{DE}$$

$$\sqrt{3} = \frac{3000\sqrt{3}}{DE}$$

$$DE = 3000 \text{ m} \quad ...(2)$$

In right ΔBEC,

$$\tan 30^o = \frac{BC}{EC}$$

$$\frac{1}{\sqrt{3}} = \frac{3000\sqrt{3}}{DE + CD}$$

$$DE + CD = 3000 \times 3$$

$$3000 + 30x = 9000$$

$$30x = 6000$$

$$x = 200 \text{ m/s}$$

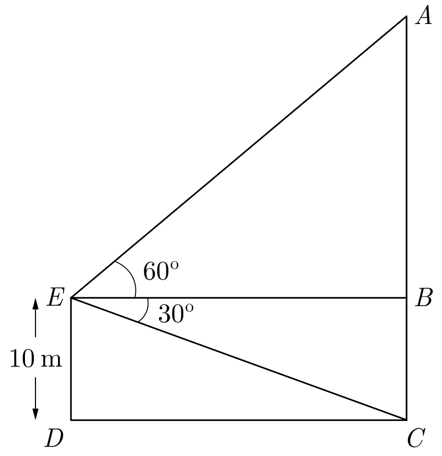
Hence, speed of plane is 200 m/s

$$= 200 \times \frac{18}{5} = 720 \text{ km/hr}$$

55. A man standing on the deck of a ship, which is 10 m above water level, observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of hill as 30°. Find the distance of the hill from the ship and the height of the hill.

Ans : [Board Term-2 OD 2016]

As per given in question we have drawn figure below. Here AC is height of hill and man is at E. $ED = 10$ is height of ship from water level.



In ΔBCE, $BC = EC = 10$ m and

$$\angle BEC = 30^{\circ}$$

Now $\tan 30^{\circ} = \frac{BC}{BE}$

$$\frac{1}{\sqrt{3}} = \frac{10}{BE}$$

$$BE = 10\sqrt{3}$$

Since $BE = CD$, distance of hill from ship

$$CD = 10\sqrt{3}\text{ m} = 10 \times 1.732 \text{ m}$$

$$= 17.32 \text{ m}$$

Now in ΔABE, $\angle AEB = 60^{\circ}$

where $AB = h$, $BE = 10\sqrt{3}$ m

and $\angle AEB = 60^{\circ}$

Thus $\tan 60^{\circ} = \frac{AB}{BE}$

$$\sqrt{3} = \frac{AB}{10\sqrt{3}}$$

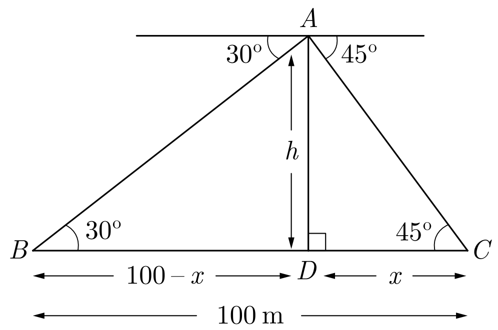
$$AB = 10\sqrt{3} \times \sqrt{3} = 30\,m$$

Thus height of hill $AB + 10 = 40$ m

56. Two ships are approaching a light house from opposite directions. The angle of depression of two ships from top of the light house are 30° and 45°. If the distance between two ships is 100 m, Find the height of light-house.

Ans : [Board Term-2 Foreign 2014]

As per given in question we have drawn figure below. Here AD is light house of height h and BC is the distance between two ships.



We have $BC = 100$ m

In ΔADC, $\tan 45^{\circ} = \frac{h}{x} \Rightarrow h = x$

In ΔABD, $\tan 30^{\circ} = \frac{h}{100 - x}$

$$\frac{1}{\sqrt{3}} = \frac{h}{100 - x}$$

$$100 - x = h\sqrt{3}$$

$$100 - h = h\sqrt{3} \qquad h = x$$

$$100 = h + h\sqrt{3}$$

$$= h(1 + \sqrt{3})$$

$$h = \frac{100}{1 + \sqrt{3}}$$

$$= \frac{100}{(\sqrt{3} + 1)} \times \frac{(\sqrt{3} - 1)}{(\sqrt{3} - 1)}$$

$$= \frac{100(\sqrt{3} - 1)}{3 - 1}$$

$$= 50(\sqrt{3} - 1)$$

$$= 50(1.732 - 1)$$

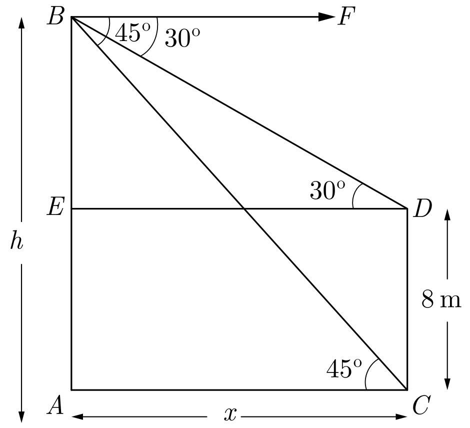
$$= 50 \times 0.732$$

Thus height of light house is 36.60 m.

57. The angles of depression of the top and bottom of an 8 m tall building from top of a multi-storeyed building are 30° and 45°, respectively. Find the height of multi-storey building and distance between two buildings.

Ans : [Board Term-2 OD 2014]

As per given in question we have drawn figure below.



Here $AE = CD = 8$ m

$$BE = AB - AE = (h - 8)$$

and $AC = DE = x$

Also, $\angle FBD = \angle BDE = 30^\circ$

$\angle FBC = \angle BCA = 45^\circ$

In right angled $\Delta\, CAB$ we have

$$\tan 45^\circ = \frac{AB}{AC}$$

$$1 = \frac{h}{x} \Rightarrow x = h \qquad ...(1)$$

In right angled $\Delta\, EDB$

$$\tan 30^\circ = \frac{BE}{ED}$$

$$\frac{1}{\sqrt{3}} = \frac{h-8}{x}$$

$$x = \sqrt{3}(h-8) \qquad ...(2)$$

From (1) and (2), we get

$$h = \sqrt{3}\,h - 8\sqrt{3}$$

$$8\sqrt{3} = \sqrt{3}\,h - h$$

$$h = \frac{8\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= 4\sqrt{3}(\sqrt{3}+1) = (12+4\sqrt{3})\text{ m}$$

Since, $x = h$, $\quad x = (12+4\sqrt{3})$

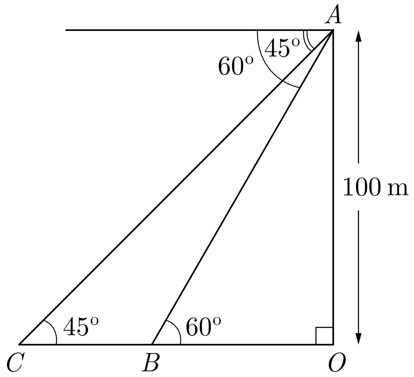
$$\text{Distance} = (12+4\sqrt{3})\text{ m}$$

Hence the height of multi storey building is $4\sqrt{3}+12$ m.

58. From a top of a building 100 m high the angle of depression of two objects are on the same side observed to be 45° and 60°. Find the distance between the objects.

Ans : [Board Term-2 OD 2014]

Let A be a point on top of building and B, C be two objects. As per given in question we have drawn figure below.



Here $\quad \angle ACO = \angle CAX = 45^\circ$

and $\quad \angle ABO = \angle XAB = 60^\circ$

In right $\Delta\, AOC$, $\frac{AO}{CO} = \tan 45^\circ$

$$\frac{100}{CO} = 1$$

$$CO = 100 \text{ m}$$

Also in right $\Delta\, AOB$, we have

$$\frac{AO}{OB} = \tan 60^\circ$$

$$\frac{100}{OB} = \sqrt{3}$$

$$OB = \frac{100}{\sqrt{3}}$$

Thus $\quad BC = CO - OB = 100 - \frac{100}{\sqrt{3}}$

$$= 100\left(1 - \frac{1}{\sqrt{3}}\right) = 100\frac{(\sqrt{3}-1)}{\sqrt{3}}$$

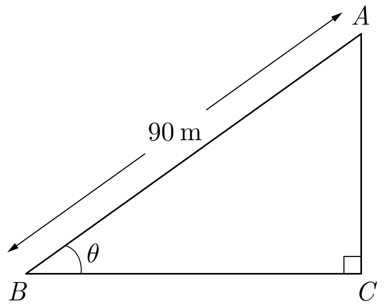
$$= 100\frac{(\sqrt{3}-1)}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{100(3-\sqrt{3})}{3}\text{ m}$$

59. A boy, flying a kite with a string of 90 m long, which is making an angle θ with the ground. Find the height of the kite. (Given $\tan\theta = \frac{45}{8}$)

Ans : [Board Term-2 OD 2014]

Let A be the position of kite and AB be the string. As per given in question we have drawn figure below.



Since $$\tan\theta = \frac{15}{8} = \frac{AC}{BC} = k$$

Let AC be $15k$ and BC be $8k$. Now using Pythagoras Theorem

$$AB = \sqrt{BC^2 + AC^2}$$
$$= \sqrt{(15k)^2 + (8k)^2} = 17k$$

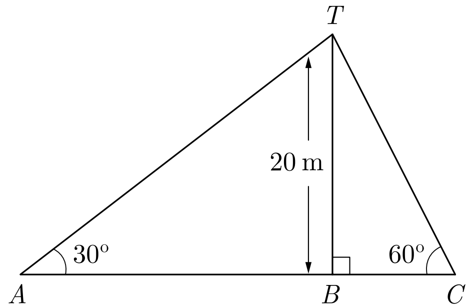
In ΔACB, $$\frac{AC}{AB} = \sin\theta$$
$$\frac{AC}{90} = \frac{15k}{17k} = \frac{15}{17}$$
$$AC = \frac{15\times 90}{17} = 79.41 \text{ m}$$

Hence, height of kite is 79.41 m.

60. Two men standing on opposite sides of a tower measure the angles of elevation of he top of the tower as 30° and 60° respectively. If the height of the tower in 20 m, then find the distance between the two men.

Ans : [Board Term-2 OD 2013]

Let two men are standing at A and C and BT is the tower. As per given in question we have drawn figure below.



In right angle triangle ΔABT,

$$\tan 30^\circ = \frac{BT}{AB}$$
$$\frac{1}{\sqrt{3}} = \frac{20}{AB}$$
$$AB = \sqrt{3}\,20$$

In right angle triangle ΔTBC,

$$\tan 60^\circ = \frac{BT}{BC}$$
$$\sqrt{3} = \frac{20}{BC}$$
$$BC = \frac{20}{\sqrt{3}}$$

Thus distance between two men,

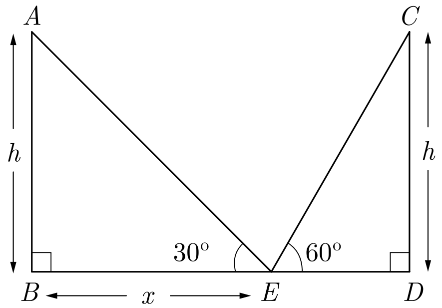
$$AB + BC = 20\sqrt{3} + \frac{20}{\sqrt{3}} = \frac{60+20}{\sqrt{3}} = \frac{80\sqrt{3}}{3} \text{ m.}$$

Hence, distance between the men is $\frac{80\sqrt{3}}{3}$ m.

61. Two poles of equal heights are standing opposite to each other on either side of a road, which is 80 m wide. From a point between them on the road, angles of elevation of their top are 30° and 60°. Find the height of the poles and distance of point from poles.

Ans : [Board 2019 Delhi Std, OD 2011]

Let the distance between pole AB and man E be x. As per given in question we have drawn figure below.



Here distance between pole CD and man is $80 - x$.

In right angle triangle ΔABE,

$$\tan 30^\circ = \frac{h}{x}$$
$$h = \frac{x}{\sqrt{3}} \qquad ...(1)$$

In angle triangle ΔCDE,

$$\tan 60^\circ = \frac{h}{80-x}$$

$$\sqrt{3} = \frac{h}{80-x}$$

$$h = 80\sqrt{3} - x\sqrt{3} \qquad ...(2)$$

Comparing (1) and (2) we have

$$\frac{x}{\sqrt{3}} = 80\sqrt{3} - x\sqrt{3}$$

$$x = 80 \times 3 - x \times$$

$$4x = 240$$

$$x = \frac{240}{4} = 60 \text{ m}$$

Substituting this value of x in (1) we have

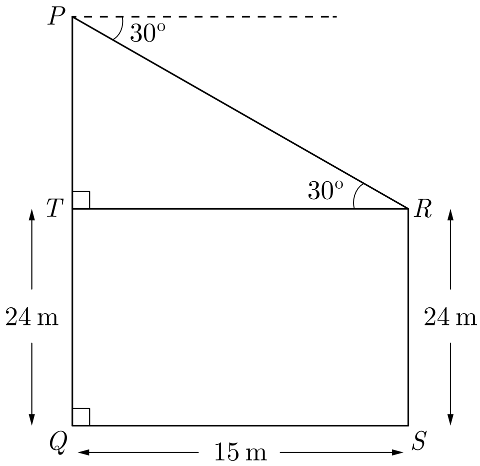
$$h = \frac{60}{\sqrt{3}} = 20\sqrt{3}$$

Hence, height of the pole is 34.64 m

62. The horizontal distance between two poles is 15 m. The angle of depression of the top of first pole as seen from the top of second pole is 30°. If the height of the first of the pole is 24 m, find the height of the second pole. [Use $\sqrt{3} = 1.732$]

Ans : [Board Term-2 2013]

Let RS be first pole and PQ be second pole. As per given in question we have drawn figure below.



In right ΔPTR,

$$\tan 30^\circ = \frac{PT}{TR}$$

$$\frac{1}{\sqrt{3}} = \frac{PT}{15}$$

$$PT = \frac{15}{\sqrt{3}} = 5\sqrt{3}$$

$$= 5 \times 1.732 = 8.66$$

$$PQ = PT + TQ$$

$$= 8.66 + 24$$

$$= 32.66 \text{ m}$$

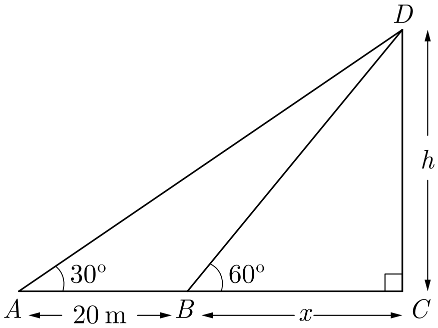
Thus height of the second pole is 32.66 m.

63. The angle of elevation of the top of a tower from a point A on the ground is 30°. On moving a distance of 20 metre towards the foot of the tower to a point B the angle of elevation increase to 60°. Find the height of the tower and the distance of the tower from the point A.

Ans : [Board Term-2 2012]

Let height of tower CD be h and distance BC be x.

As per given in question we have drawn figure below.



In right ΔDBC, $\frac{h}{x} = \tan 60^\circ$

$$h = \sqrt{3}\,x \qquad ...(1)$$

In right ΔADC,

$$\frac{h}{x+20} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\sqrt{3}\,h = x + 20 \qquad ...(2)$$

Substituting the value of h from eq. (1) in eq. (2), we get

$$3x = x + 20$$

$$x = 10 \text{ m} \qquad ...(3)$$

Thus $AC = 20 + x = = 30$ m.

and $h = \sqrt{3} \times 10 = 10\sqrt{3}$

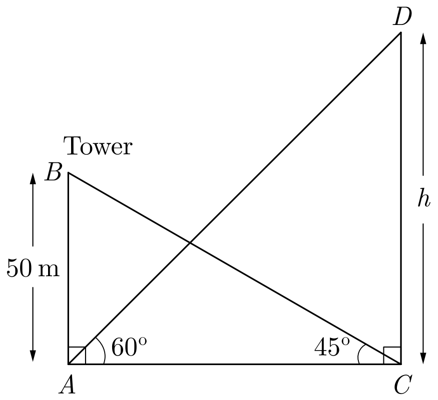
$$= 10 \times 1.732 = 17.32 \text{ m}$$

Hence, height of tower is 17.32 m and distance of tower from point A is 30 m.

64. The angle of elevation of the top of a hill at the foot of a tower is 60° and the angle of elevation of the top of the tower from the foot of the hill is 30°. If the tower is 50 m high, find the height of the hill.

Ans : [Board Term-2 2012]

Let AB be tower of height of 50 m and DC be hill of height h. As per given in question we have drawn figure below.



In right ΔBAC,

$$\cos 30^\circ = \frac{AC}{50}$$

$$\sqrt{3} = \frac{AC}{50}$$

$$AC = 50\sqrt{3}$$

In right ΔACD,

$$\tan 60^\circ = \frac{CD}{50\sqrt{3}}$$

$$\sqrt{3} = \frac{CD}{50\sqrt{3}}$$

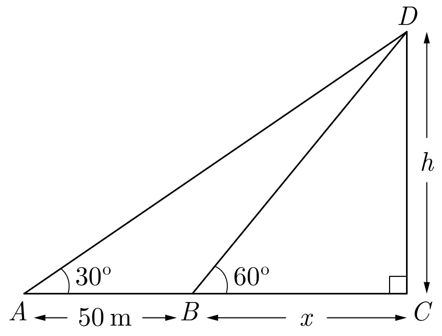
$$CD = 50\sqrt{3} \times \sqrt{3} = 150 \text{ m}$$

Thus height of the hill $CD = 150$ m

65. A person observed the angle of elevation of the top of a tower as 30°. He walked 50 m towards the foot of the tower along level ground and found the angle of elevation of the top of the tower as 60°. Find the height of the tower.

Ans : [Board Term-2 2012]

Let DC be tower of height h. As per given in question we have drawn figure below.



Here A is the point at elevation 30° and B is the point of elevation at 60°.

Let BC be x.

Now $$AC = (50 + x) \text{ m}$$

In right ΔDCB, $$\frac{h}{x} = \tan 60^\circ = \sqrt{3}$$

$$h = \sqrt{3}\,x \qquad ...(1)$$

In right ΔDCA,

$$\frac{h}{x+50} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\sqrt{3}\,h = x + 50 \qquad (1)$$

Substituting the value of h from (1) in (2), we have

$$3x = x + 50$$

$$2x = 50 \Rightarrow x = 25 \text{ m}$$

$$h = 25\sqrt{3}$$

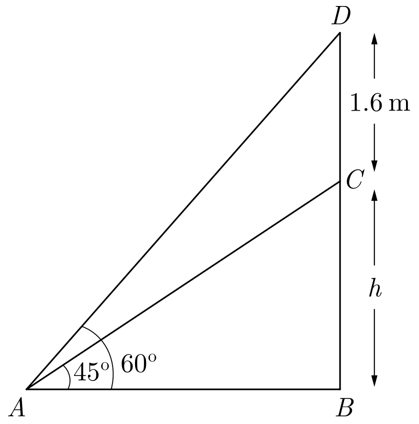
$$= 25 \times 1.732 = 43.3 \text{ m}$$

Hence height of tower is 43.3 m.

66. A statue 1.6 m tall stands on the top of a pedestal. From a point on the ground the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal.

Ans : [Board Term-2 OD 2012]

Let CD be statue of 1.6 m and pedestal BC of height h. Let A be point on ground. As per given in question we have drawn figure below.



In right ΔABD,

$$\cot 60^\circ = \frac{AB}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{h+1.6}$$

$$AB = \frac{h+1.6}{\sqrt{3}} \qquad ...(1)$$

In right ΔABC,

$$\frac{AB}{BC} = \cot 45^\circ$$

$$1 = \frac{AB}{h}$$

$$AB = h \qquad ...(2)$$

From (1) and (2), we get

$$h = \frac{h+1.6}{\sqrt{3}}$$

$$h\sqrt{3} = h+1.6$$

$$h\sqrt{3} - h = 1.6$$

$$h(\sqrt{3}-1) = 1.6$$

$$h = \frac{1.6}{\sqrt{3}-1} = \frac{1.6}{1.732-1}$$

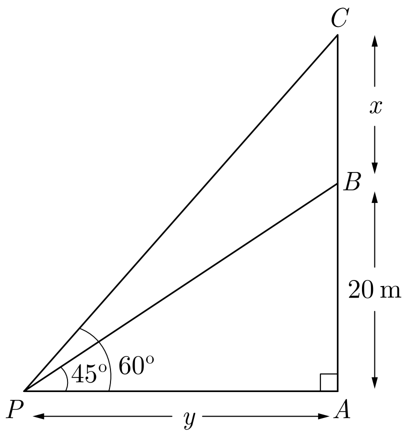
$$= \frac{1.6}{0.732} = 2.185 \text{ m}$$

Height of pedestal h is 2.2 m.

67. From a point on a ground, the angle of elevation of bottom and top a transmission tower fixed on the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.

Ans : [Board Term-2 OD Compt. 2017]

Let P be the point on ground, AB be the building of height 20 m and BC be the tower of height x. As per given in question we have drawn figure below.



In right ΔBAP we have

$$\frac{BA}{PA} = \tan 45^\circ$$

$$\frac{20}{y} = 1$$

$$y = 20$$

In right ΔCAP,

$$\frac{CA}{PA} = \tan 60^\circ$$

$$\frac{20+x}{y} = \sqrt{3}$$

$$20 + x = y\sqrt{3}$$

$$20 + x = 20\sqrt{3}$$

$$x = 20\sqrt{3} - 20$$

$$= 20(\sqrt{3}-1)$$

$$= 20 \times (1.732 - 1)$$

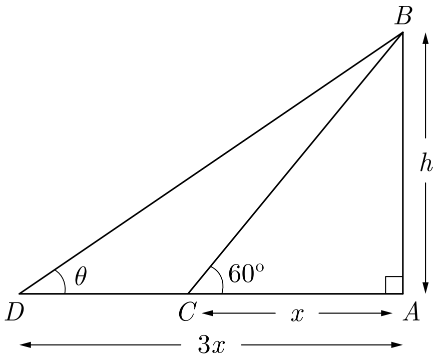
$$= 20 \times 0.73 = 14.64$$

Hence, height of the tower is 14.64 m.

68. The shadow of a tower at a time is three times as long as its shadow when the angle of elevation of the sun is 60°. Find the angle of elevation of the sun at the of the longer shadow.

Ans : [Board Term-2 Foreign 2017]

Let AB be tower of height h, AC be the shadow at elevation of sun of 60°. As per given in question we have drawn figure below.



In right $\Delta\, BAC$,

$$\frac{AB}{AC} = \tan 60^\circ$$

$$\frac{h}{x} = \sqrt{3}$$

$$h = x\sqrt{3}$$

In right $\Delta\, BAD$,

$$\frac{AB}{AD} = \tan\theta$$

$$\frac{h}{3x} = \tan\theta$$

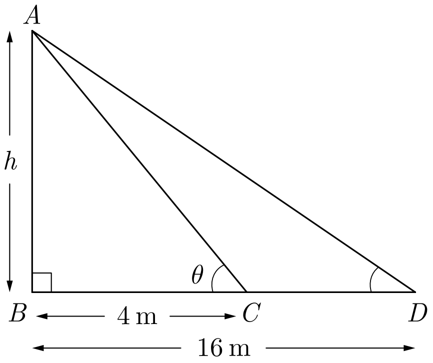
$$\frac{x\sqrt{3}}{3x} = \frac{1}{\sqrt{3}} = \tan 30^\circ$$

Thus $\theta = 30^\circ$.

69. On a straight line passing through the foot of a tower, two C and D are at distance of 4 m and 16 m from the foot respectively. If the angles of elevation from C and D of the top of the tower are complementary, then find the height of the tower.

Ans : [Board Term-2 OD 2017]

Let AB be tower of height h, C and D be the two point. As per given in question we have drawn figure below.



Since $\angle ACB$ and $\angle ADB$ are complementary,

$$\angle ACB = \theta \text{ and } \angle ADB = 90^\circ - \theta$$

Now, in right $\Delta\, ABC$,

$$\tan\theta = \frac{AB}{BC} = \frac{h}{4} \quad ...(1)$$

In right $\Delta\, ABD$,

$$\tan(90 - \theta) = \frac{AB}{BD} = \frac{h}{16}$$

$$\cot\theta = \frac{h}{16}$$

$$\tan\theta = \frac{16}{h} \quad ...(2)$$

From (1) and (2) we have

$$\frac{h}{4} = \frac{16}{h}$$

$$h^2 = 4 \times 16 = 64 = 8^2 \Rightarrow h = 8 \text{ m}$$

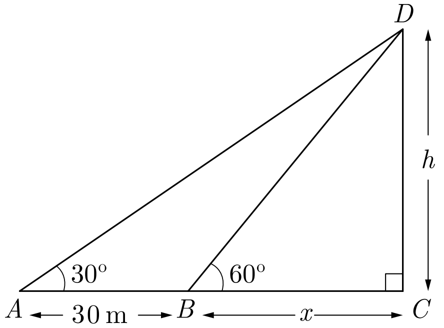
Thus height of tower is 8 m.

FOUR MARKS QUESTIONS

70. The person standing on the bank of river observes that the angle of elevation of the top of a tree standing on opposite bank is 60°. When he moves 30 m away from the bank, he finds the angle of elevation to be 30°. Find the height of tree and width of the river.

Ans : [Board 2020 OD Basic]

Let CD be the tree of height h. Let A be the position of person after moving 30 m away from point B on bank of river. Let $BC = x$ be the width of the river. As per given in question we have drawn figure below.



In right ΔDBC, $\frac{h}{x} = \tan 60°$

$$h = \sqrt{3}\,x \qquad ...(1)$$

In right ΔADC,

$$\frac{h}{x+30} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\sqrt{3}\,h = x+30 \qquad ...(2)$$

Substituting the value of h from eq. (1) in eq. (2), we get

$$3x = x+30$$

$$x = 15 \text{ m} \qquad ...(3)$$

Thus $h = \sqrt{3} \times 15 = 15\sqrt{3}$

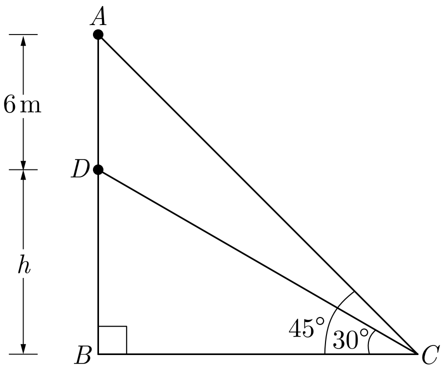
$$= 15 \times 1.732 = 25.98 \text{ m}$$

Hence, height of tree is 25.98 m and width of river is 15 m.

71. A vertical tower stands on horizontal plane and is surmounted by a vertical flag-staff of height 6 m. At a point on the ground, angle of elevation of the bottom and top of the flag-staff are 30° and 45° respectively. Find the height of the tower. (Take $\sqrt{3} = 1.73$)

Ans : [Board 2020 Delhi Standard]

From the given information we have drawn the figure as below.



Here AD is a flagstaff and BD is a tower.

In ΔABC $\tan 45° = \frac{AB}{BC}$

$$1 = \frac{h+6}{BC}$$

$$BC = h+6 \qquad ...(1)$$

In ΔDBC, $\tan 30° = \frac{DB}{BC}$ from (1)

$$\frac{1}{\sqrt{3}} = \frac{h}{h+6}$$

$$h\sqrt{3} = h+6$$

$$h(\sqrt{3}-1) = 6$$

$$h = \frac{6}{\sqrt{3}-1}$$

$$= \frac{6}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \frac{6(\sqrt{3}+1)}{2}$$

$$= 3(\sqrt{3}+1)$$

$$= 3(1.73+1)$$

$$= 3 \times 2.73$$

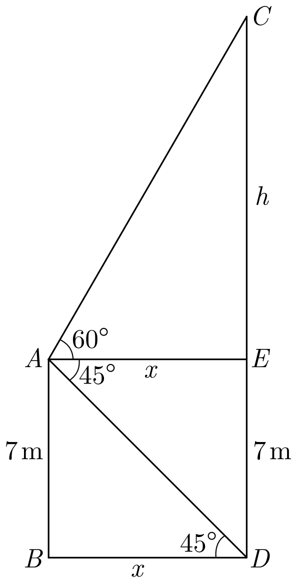
$$= 8.19 \text{ m}$$

Thus height of tower is 8.19 m.

72. From the top of a 7 m high building the angle of elevation of the top of a tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

Ans : [Board 2020 Delhi Standard]

Let AB be a building of height 7 m and CD be tower of height CD. From the given information we have drawn the figure as below.



Now $CD = (7+h)$

$BD = AE = x$

In $\Delta\,ABD$, $\tan 45^\circ = \dfrac{AB}{BD}$

$$1 = \frac{7}{x} \Rightarrow x = 7\,\text{cm}$$

In $\Delta\,CEA$, $\tan 60^\circ = \dfrac{CE}{AE}$

$$\sqrt{3} = \frac{h}{x} \Rightarrow h = x\sqrt{3}$$

Substituting the value of x, we get

$$h = 7\sqrt{3}$$

Now, $CD = CE + ED$

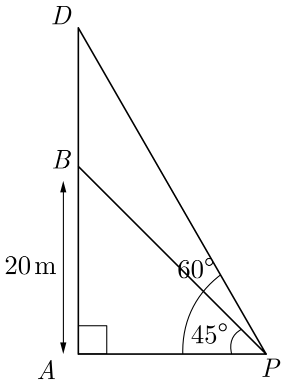
$$= (7+7\sqrt{3})\text{ m}$$
$$= 7(1+\sqrt{3})\text{ m}$$
$$= 7(1+1.732)\text{ m}$$
$$= 7\times 2.732\text{ m}$$
$$= 19.124\text{ m}$$

Hence height of tower is 19.12 m approximately.

73. From a point on the ground, the angles of elevation of the bottom and the top of a tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.

Ans : [Board 2020 OD Standard]

As per given information in question we have drawn the figure below. Here AB is the building and BD is tower on building.



In $\Delta\,PAB$, $\tan 45^\circ = \dfrac{AB}{AP}$

$$1 = \frac{20}{AP} \Rightarrow AP = 20\text{ m}$$

In $\Delta\,PAD$, $\tan 60^\circ = \dfrac{AD}{AP} = \dfrac{20+BD}{20}$

$$\sqrt{3} = \frac{20+BD}{20}$$

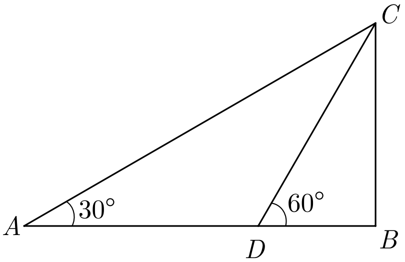
$$20 + BD = 20\sqrt{3}$$

$$BD = 20\sqrt{3} - 20 = 20(\sqrt{3}-1)$$
$$= 20(1.732-1)$$
$$= 20\times 0.732$$
$$= 14.64\text{ cm}.$$

74. A man in a boat rowing away from a light house 100 m high takes 2 minutes to change the angle of elevation of the top of the light house from 60° to 30°. Find the speed of the boat in metres per minute. [Use $\sqrt{3} = 1.732$]

Ans : [Board 2019 Delhi Standard]

As per given information in question we have drawn the figure below.



Here D is first position and A is position after 2 minutes.

Height of the light house,

$$BC = 100\text{ m}$$

From $\Delta\,DBC$, $\angle B = 90^\circ$

So, $\tan 60^\circ = \dfrac{BC}{BD}$

$$\sqrt{3} = \frac{100}{BD}$$

$$BD = \frac{100}{\sqrt{3}}\,\text{m}$$

Now, after time 2 minute boat is at A. New distance from light house is AB and angle is 30°.

From $\Delta\, ABC$, $\angle B = 90^\circ$

So, $\tan 30^\circ = \dfrac{BC}{AB}$

$$\frac{1}{\sqrt{3}} = \frac{100}{AB}$$

$$AB = 100\sqrt{3}$$

Therefore, distance d travelled in 2 min,

$$AD = AB - DB = 100\sqrt{3} - \frac{100}{3}$$

$$= 173.2 - \frac{100}{3}\sqrt{3}$$

$$= 173.2 - 57.73 = 115.47 \text{ m}$$

Speed $s = \dfrac{d}{t} = \dfrac{115.47\text{ m}}{2\text{ min}}$

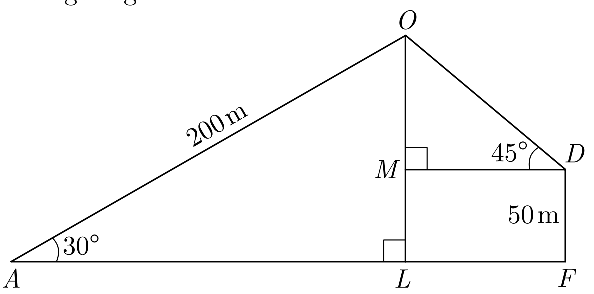
$$= 57.74\,\text{m/min}$$

Hence, going away from the light house with a speed of $57.74\,\text{m/min}$.

75. Amit, standing on a horizontal plane, find a bird flying at a distance of 200 m from him at an elevation of 30°. Deepak standing on the roof of a 50 m high building, find the angle of elevation of the same bird to be 45°. Amit and Deepak are on opposite sides of the bird. Find the distance of the bird from Deepak.

Ans : [Board 2019 OD Standard]

As per given information in question we have drawn the figure given below.



Let O be the position of the bird, A be the position for Amit, D be the position for Deepak and FD be the building at which Deepak is standing at height 50 m.

In $\Delta\, OLA$, $\angle L = 90^\circ$

$$\sin 30^\circ = \frac{OL}{OA}$$

$$\frac{1}{2} = \frac{OL}{200} \Rightarrow OL = \frac{200}{2} = 100\,\text{m}$$

$$OM = OL - LM$$

$$= OL - FD$$

$$= (100 - 50)\,\text{m} = 50\,\text{m}$$

In $\Delta\, OMD$, $\angle M = 90^\circ$

$$\sin 45^\circ = \frac{OM}{OD}$$

$$\frac{1}{\sqrt{2}} = \frac{50}{OD}$$

$$OD = 50\sqrt{2}$$

$$= 50 \times 1.414 = 70.7\,\text{m}$$

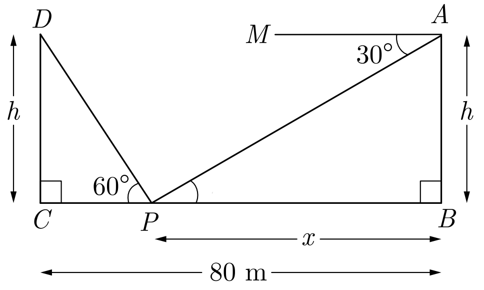
Thus, the distance of the bird from the Deepak is 70.7 m.

With

76. Two poles of equal heights are standing opposite to each other on either side of the road which is 80 m wide. From a point P between them on the road, the angle of elevation of the top of a pole of a pole is 60° and the angle of depression from the top of the other pole of point P is 30°. Find the heights of the poles and the distance of the point P from the poles.

Ans : [Board 2019 OD Standard]

Let the distance between pole AB and point P be x. As per given in question we have drawn figure below.



Here distance between pole CD and P is $80 - x$.

In right angle triangle $\Delta\, ABP$, $\angle APB = 30^\circ$

$$\tan 30^\circ = \frac{h}{x}$$

$$h = \frac{x}{\sqrt{3}} \qquad ...(1)$$

In angle triangle $\Delta\, CDP$,

$$\tan 60^\circ = \frac{CD}{CP} = \frac{CD}{CB - PB}$$

$$\sqrt{3} = \frac{h}{80 - x}$$

$$h = 80\sqrt{3} - x\sqrt{3} \qquad ...(2)$$

Comparing (1) and (2) we have

$$\frac{x}{\sqrt{3}} = 80\sqrt{3} - x\sqrt{3}$$

$$x = 80 \times 3 - x \times 3$$

$$4x = 240$$

$$x = \frac{240}{4} = 60 \text{ m}$$

Substituting this value of x in (1) we have

$$h = \frac{60}{\sqrt{3}} = 20\sqrt{3} = 34.64 \text{ m}$$

Hence, height of the pole AB and CD is 34.64 m

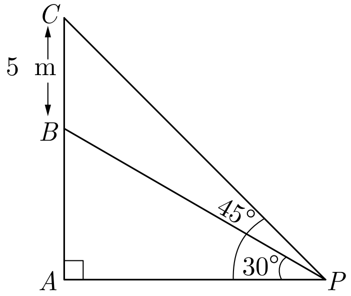
Distance of point P from pole AB is 20 m.

Distance of point P from pole CD is 60 m.

77. From a point P on the ground, the angle of elevation of the top of a tower is 30° and that of the top of the flagstaff is 45°. If height of flagstaff is 5 m, find the height of the tower. (Use $\sqrt{3} = 1.732$)

Ans : [Board 2019 OD Standard]

Let AB denotes the height of the tower and BC denotes the height of the flag. As per given information in question we have drawn the figure as given below.



From $\Delta\, BAP$, $\angle A = 90^\circ$

Now, $$\tan 30^\circ = \frac{AB}{AP}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{AP}$$

$$AP = \sqrt{3}\, AB \qquad ...(1)$$

Again from $\Delta\, CAP$,

$$\angle A = 90^\circ$$

and $$\tan 45^\circ = \frac{AC}{AP}$$

$$1 = \frac{AC}{AP}$$

$$AP = AC = (AB + BC)$$

$$AP = (AB + 5) \qquad ...(2)$$

From equation (1) and (2), we obtain,

$$(AB + 5) = \sqrt{3}\, AB$$

$$5 = \sqrt{3}\, AB - AB$$

$$AB = \frac{5}{(\sqrt{3} - 1)} = \frac{5}{(1.732 - 1)}$$

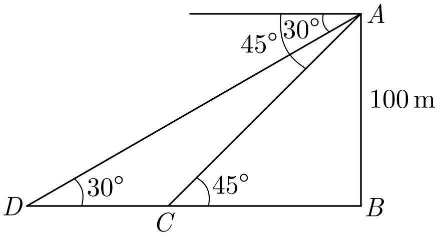
$$= \frac{5}{0.732} = 6.8306 \text{ m.}$$

Hence, height of the tower, $AB = 6.8306$ m.

78. As observed from the top of a 100 m high light house from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the light house, find the distance between the two ships [Use $\sqrt{3} = 1.732$]

Ans : [Board 2018]

Let AB be the tower and ships are at points C and D. As per question statement we have shown digram below.



Now in ΔABC we have

$$\tan 45^\circ = \frac{AB}{AC}$$

$$\frac{AB}{AC} = 1 \Rightarrow AB = BC$$

Now in ΔABD we have

$$\tan 30^\circ = \frac{AB}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{BC+CD}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{AB+CD}$$

$$AB+CD = \sqrt{3}\,AB$$

$$CD = AB(\sqrt{3}-1)$$

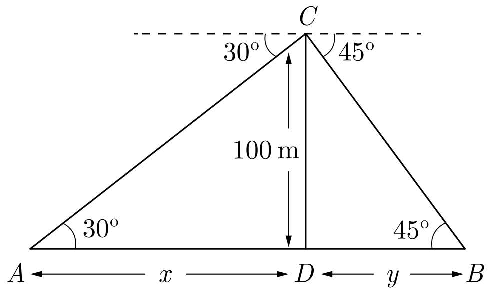
$$= 100 \times (1.732-1)$$

$$= 73.2\,\text{m}$$

79. Distance between two ships is 73.2 m. From the top of tower, 100 m high, a man observes two cars on the opposite sides of the tower with the angles of depression 30° and 45° respectively. Find the distance between the cars. (Use $\sqrt{3} = 1.73$)

Ans : [Board Term-2 SQP 2016]

Let DC be tower of height 100 m. A and B be two car on the opposite side of tower. As per given in question we have drawn figure below.



In right $\Delta\, ADC$,

$$\tan 30^\circ = \frac{CD}{AD}$$

$$\frac{1}{\sqrt{3}} = \frac{100}{x}$$

$$x = 100\sqrt{3} \qquad ...(1)$$

In right $\Delta\, BDC$,

$$\tan 45^\circ = \frac{CD}{DB}$$

$$1 = \frac{100}{y} \Rightarrow y = 100 \text{ m}$$

Distance between two cars

$$AB = AD + DB = x + y$$

$$= (100\sqrt{3} + 100)$$

$$= (100 \times 1.73 + 100) \text{ m}$$

$$= (173 + 100) \text{ m}$$

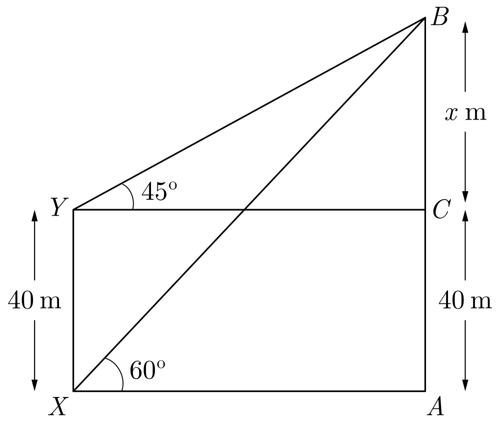
$$= 273 \text{ m}$$

Hence, distance between two cars is 273 m.

80. The angle of elevation of the top B of a tower AB from a point X on the ground is 60°. At point Y, 40 m vertically above X, the angle of elevation of the top is 45°. Find the height of the tower AB and the distance XB.

Ans : [Board Term-2 OD 2016]

As per given in question we have drawn figure below.



In right $\Delta\, YCB$, we have

$$\tan 45^\circ = \frac{BC}{YC}$$

$$1 = \frac{x}{YC}$$

$$YC = x = XA$$

In right $\Delta\, XAB$ we have

$$\tan 60^\circ = \frac{AB}{XA}$$

$$\sqrt{3} = \frac{x+40}{x}$$

$$\sqrt{3}\,x = x + 40$$

$$x\sqrt{3} - x = 40$$

$$x = \frac{40}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= 20(\sqrt{3}+1)$$

$$= 20\sqrt{3} + 20$$

Thus height of the tower,

$$AB = x + 40$$
$$= 20\sqrt{3} + 20 + 40$$
$$= 20\sqrt{3} + 60$$
$$= 20(\sqrt{3} + 3)$$

In right ΔXAB we have,

$$\sin 60^\circ = \frac{AB}{BX}$$

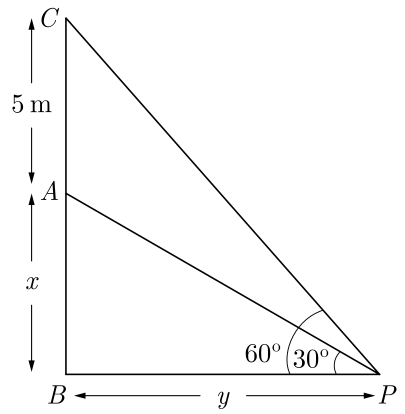
$$\frac{\sqrt{3}}{2} = \frac{AB}{BX}$$

$$BX = \frac{2AB}{\sqrt{3}} = \frac{20 \times 2(\sqrt{3}+3)}{\sqrt{3}}$$
$$= 40(1 + \sqrt{3})$$
$$= 40 \times 2.73 = 109.20$$

81. A vertical tower stands on a horizontal plane and is surmounted by a flagstaff of height 5 m. From a point on the ground the angles of elevation of top and bottom of the flagstaff are 60° and 30° respectively. Find the height of the tower and the distance of the point from the tower. (take $\sqrt{3} = 1.732$)

Ans : [Board Term-2 Foreign Set I, 2016]

Let AB be tower of height x and AC be flag staff of height 5 m. As per given in question we have drawn figure below.



In right ΔABP,

$$\frac{AB}{BP} = \tan 30^\circ$$

$$\frac{x}{y} = \frac{1}{\sqrt{3}}$$

$$y = \sqrt{3}\,x \qquad ...(1)$$

In right ΔCBP

$$\frac{x+5}{y} = \tan 60^\circ = \sqrt{3} \qquad ...(2)$$

Substituting the value of y from (1) we have

$$\frac{x+5}{\sqrt{3}\,x} = \sqrt{3}$$

$$x + 5 = 3x \Rightarrow x = 2.5 \text{ m}$$

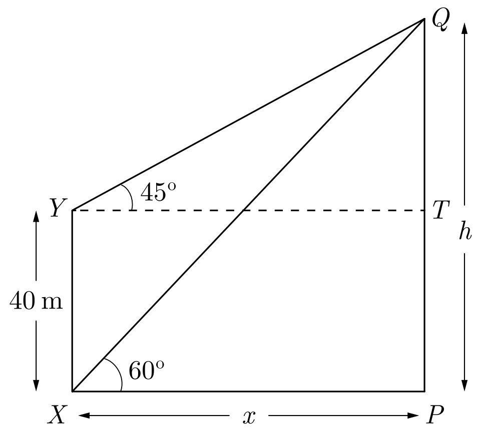
Height of tower is = 2.5 m

Distance of P from tower $= (2.5 \times 1.732)$ or 4.33 m.

82. The angle of elevation of the top Q of a vertical tower PQ from a point X on the ground is 60°. From a point Y 40 m vertically above X, the angle of elevation of the top Q of tower is 45°. Find the height of the PQ and the distance PX. (Use $\sqrt{3} = 1.73$)

Ans : [Board Term-2 OD 2015]

Let PX be x and PQ be h. As per given in question we have drawn figure below.



Now $QT = (h - 40)$ m

In right ΔPQX we have,

$$\tan 60^\circ = \frac{h}{x}$$

$$\sqrt{3} = \frac{h}{x}$$

$$h = \sqrt{3}\,x \qquad ...(1)$$

In right ΔQTY we have

$$\tan 45^\circ = \frac{h - 40}{x}$$

$$1 = \frac{h-40}{x}$$

$$x = h - 40 \quad ...(2)$$

Solving (1) and (2), we get

$$x = \sqrt{3}\,x - 40$$

$$\sqrt{3}\,x - x = 40$$

$$(\sqrt{3}-1)x = 40$$

$$x = \frac{40}{\sqrt{3}-1} = 20(\sqrt{3}+1) \text{ m}$$

Thus $h = \sqrt{3} \times 20(\sqrt{3}+1)$

$$= 20(3+\sqrt{3}) \text{ m}$$

$$= 20(3+1.73)$$

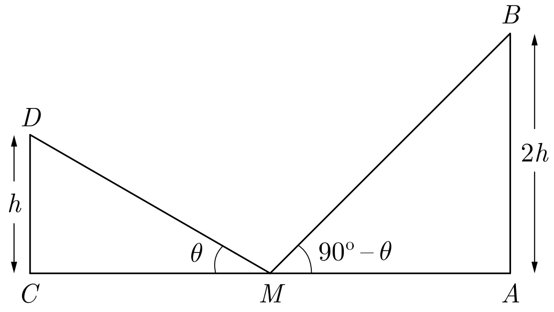
$$= 20 \times 4.73$$

Hence, height of tower is 94.6 m.

83. Two post are k metre apart and the height of one is double that of the other. If from the mid-point of the line segment joining their feet, an observer finds the angles of elevation of their tops to be complementary, then find the height of the shorted post.

Ans : [Board Term-2 Foreign 2015]

Let AB and CD be the two posts such that $AB = 2CD$. Let M be the mid-point of CA. As per given in question we have drawn figure below.



Here $CA = k$, $\angle CMD = \theta$ and $\angle AMB = 90^\circ - \theta$

Clearly, $CM = MA = \frac{1}{2}k$

Let $CD = h$. then $AB = 2h$

Now, $\frac{AB}{AM} = \tan(90^\circ - \theta)$

$$\frac{2h}{\frac{k}{2}} = \cot\theta$$

$$\frac{4h}{k} = \cot\theta \quad ...(1)$$

Also in right $\Delta\, CMD$,

$$\frac{CD}{CM} = \tan\theta$$

$$\frac{h}{\frac{k}{2}} = \tan\theta$$

$$\frac{2h}{k} = \tan\theta \quad ...(2)$$

Multiplying (1) and (2), we have

$$\frac{4h}{k} \times \frac{2h}{k} = \tan\theta \times \cot\theta = 1$$

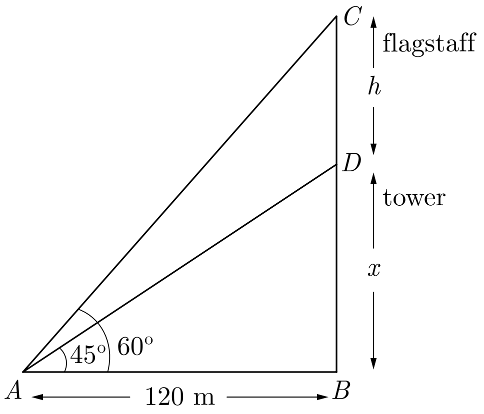
$$h^2 = \frac{k^2}{8}$$

$$h = \frac{k}{2\sqrt{2}} = \frac{k\sqrt{2}}{4}$$

84. The angle of elevation of the top of a tower at a distance of 120 m from a point A on the ground flagstaff fixed at the top of the tower, at A is 60°, then find the height of the flagstaff. [Use $\sqrt{3} = 1.73$]

Ans : [Board Term-2 OD 2014]

Let BD be the tower of height x and CD be flagstaff of height h. As per given in question we have drawn figure below.



Here $\angle DAB = 45^\circ, \angle CAB = 60^\circ$

and $AB = 120$ m

In right angled $\Delta\, ABD$ we have

$$\frac{x}{AB} = \tan 45^\circ = 1$$

$$x = AB = 120 \text{ m}$$

In right angled $\Delta\, ACB$ we have

$$\frac{h+x}{120} = \tan 60^\circ = \sqrt{3}$$

$$h + 120 = 120\sqrt{3}$$

$$h = 120\sqrt{3} - 120$$

$$= 120(\sqrt{3} - 1)$$

$$= 120(1.73 - 1)$$

$$= 120 \times 0.73$$

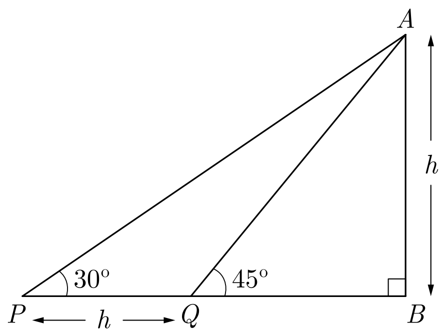
$$h = 87.6 \text{ m}$$

Hence, height of the flagstaff is 87.6 m.

85. A man on the top of a vertical tower observes a car moving at a uniform speed towards him. If it takes 12 min. for the angle of depression to change from 30° to 45°, how soon after this, the car will reach the tower ?

Ans : [Board Term-2 OD 2014]

Let AB be the tower of height h. As per given in question we have drawn figure below.



Car is at P at 30° and is at Q at 45° elevation.

Here $\angle AQB = 45^\circ$

Now, in right ΔABQ we have,

$$\tan 45^\circ = \frac{AB}{BQ}$$

$$1 = \frac{h}{BQ}$$

$$BQ = h$$

In right ΔAPB we have,

$$\tan 30^\circ = \frac{AB}{PB}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{x+h}$$

$$x + h = h\sqrt{3}$$

$$x = h(\sqrt{3} - 1)$$

Thus, $$\text{Speed} = \frac{h(\sqrt{3}-1)}{12} \text{ m/min}$$

Time for remaining distance,

$$t = \frac{h}{\frac{h(\sqrt{3}-1)}{12}} = \frac{12}{(\sqrt{3}-1)}$$

$$= \frac{12(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{12(\sqrt{3}+1)}{3-1}$$

$$= \frac{12}{2}(\sqrt{3}+1)$$

$$= 6(\sqrt{3}+1)$$

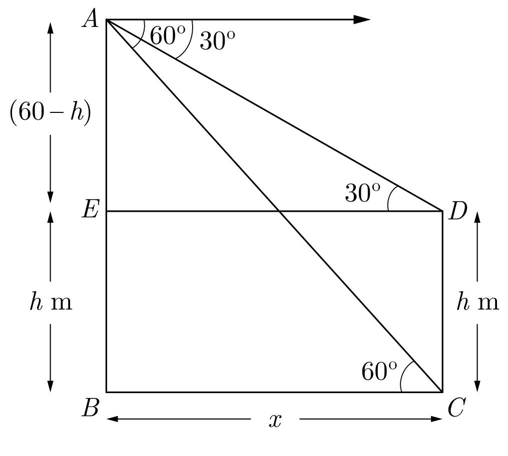
$$t = 6 \times 2.73 = 16.38$$

Hence, time taken by car is 16.38 minutes.

86. From the top of a building 60 m high the angles of depression of the top and the bottom of a tower are observed to be 30° and 60°. Find the height of the tower.

Ans : [Board Term-2 2011, 2012, OD 2014]

Let AB be the building of height 60 m and CD be the tower of height h. Angle of depressions of top and bottom are given 30° and 60° respectively. As per given in question we have drawn figure below.



Here $DC = EB = h$ and let $BC = x$

$$AE = (60 - h) \text{ m}$$

In right angled ΔAED we have

$$\frac{60-h}{ED} = \tan 30^\circ$$

$$\frac{60-h}{x} = \frac{1}{\sqrt{3}}$$

$$\sqrt{3}(60-h) = x \quad ...(1)$$

In right ΔABC we have

$$\frac{60}{x} = \tan 60^\circ$$

$$60 = \sqrt{3}\,x \quad ...(2)$$

Substituting the value of x from equation (1) in equation (2), we have

$$60 = \sqrt{3} \times \sqrt{3}(60-h)$$

$$60 = 3 \times (60-h)$$

$$20 = 60-h$$

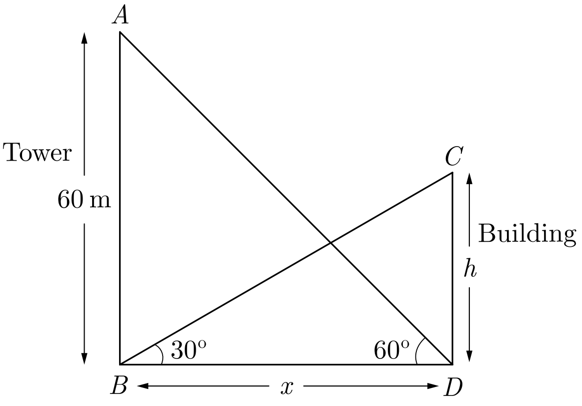
$$h = 40 \text{ m}$$

Hence, height of tower is 40 m.

87. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 60 m high, find the height of the building.

Ans : [Board 2020 Delhi Basic, Delhi 2013]

Let AB be the tower of 60 m height and CD be the building of h height. As per given in question we have drawn figure below.



In right ΔABD we have

$$\tan 60^\circ = \frac{AB}{BD}$$

$$\sqrt{3} = \frac{60}{x}$$

$$x = \frac{60}{\sqrt{3}} = 20\sqrt{3}$$

Now, in right ΔBCD we have

$$\tan 30^\circ = \frac{CD}{BD} = \frac{h}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{20\sqrt{3}}$$

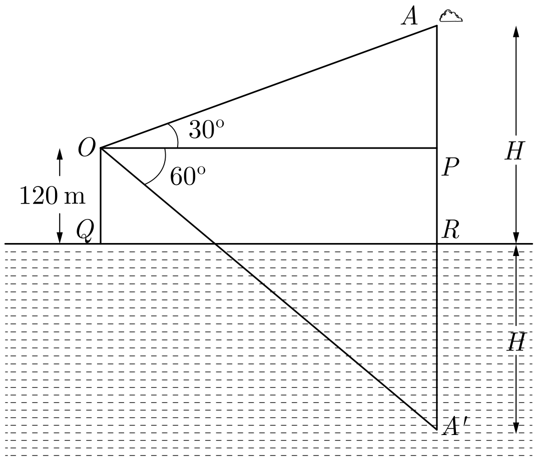
$$h = \frac{20\sqrt{3}}{\sqrt{3}} = 20$$

Hence height of the building is 20 m.

88. The angle of elevation of a cloud from a point 120 m above a lake is 30° and the angle of depression of its reflection in the lake is 60°. Find the height of the cloud.

Ans : [Board Term-2 OD 2012]

As per given in question we have drawn figure below.



Here A is cloud and A' is refection of cloud.

In right ΔAOP we have

$$\tan 30^\circ = \frac{PA}{OP}$$

$$\frac{1}{\sqrt{3}} = \frac{H-120}{OP}$$

$$OP = (H-120)\sqrt{3} \quad ...(1)$$

In right $\Delta OPA'$ we have

$$\tan 60^\circ = \frac{PA'}{OP}$$

$$\sqrt{3} = \frac{H+120}{OP}$$

$$OP = \frac{H+120}{\sqrt{3}} \qquad ...(2)$$

From (1) and (2), we get

$$\frac{H+120}{\sqrt{3}} = \sqrt{3}(H-120)$$

$$H+120 = 3(H-120)$$

$$H+120 = 3H-360$$

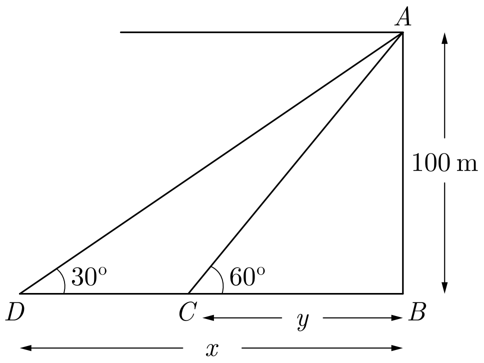
$$2H = 480 \Rightarrow H = 240$$

Thus height of cloud is 240 m.

89. As observed from the top of a light house, 100 m high above sea level, the angles of depression of a ship, sailing directly towards it, changes from 30° to 60°. Find the distance travelled by the ship during the period of observation. (Use $\sqrt{3} = 1.73$)

Ans : [Board Term-2 OD 2016]

Let AB be the light house of height 100 m. Let C and D be the position of ship at elevation 60° and 30°. As per given in question we have drawn figure below.



In right $\Delta\, ABC$ we have

$$\frac{AB}{BC} = \tan 60^\circ$$

$$\frac{100}{y} = \sqrt{3}$$

$$y = \frac{100}{\sqrt{3}}$$

In right $\Delta\, ABD$, we have

$$\frac{AB}{BD} = \tan 30^\circ$$

$$\frac{100}{x} = \frac{1}{\sqrt{3}}$$

$$x = 100\sqrt{3}$$

Distance CD travelled by ship,

$$x-y = 100\sqrt{3} - \frac{100}{\sqrt{3}} \text{ m}$$

$$= 100\left[\frac{3-1}{\sqrt{3}}\right]$$

$$= \frac{100\times 2}{\sqrt{3}}$$

$$= \frac{200}{\sqrt{3}} = \frac{200\sqrt{3}}{3}$$

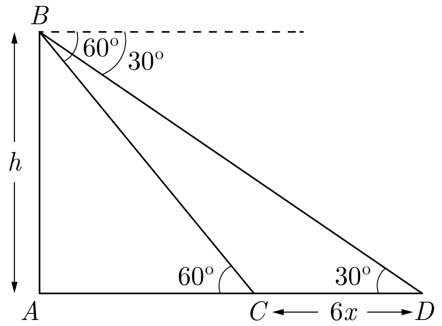
$$= \frac{200\times 1.73}{3} = \frac{3.46}{3} \text{ m}$$

$$= 115.33 \text{ m}$$

90. A straight highway leads to the foot of a tower. A man standing on its top observes a car at an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. 6 seconds later, the angle of depression of the car becomes 60°. Find the time taken by the car to reach the foot of tower from this point.

Ans : [Board Term-2 Delhi Compt. 2017]

Let AB be the tower of height h. Let point C and D be location of car. As per given in question we have drawn figure below.



Let the speed of car be x.
Thus distance covered in 6 sec $= 6x$.

Hence $DC = 6x$

Let distance (remaining) CA covered in t sec.

$$CA = tx$$

Now in right $\Delta\, ADB$,

$$AD = AC + CD = 6x + tx$$

$$\tan 30^\circ = \frac{h}{6x+tx}$$

$$\frac{h}{x} = \frac{6+t}{\sqrt{3}} \quad ...(1)$$

In right ΔACB we have,

$$\tan 60^\circ = \frac{h}{tx}$$

$$\sqrt{3}\,t = \frac{h}{x} \quad ...(2)$$

From eqn. (1) and (2) we get

$$\sqrt{3}\,t = \frac{6+t}{\sqrt{3}}$$

$$3t = 6+t$$

$$2t = 6$$

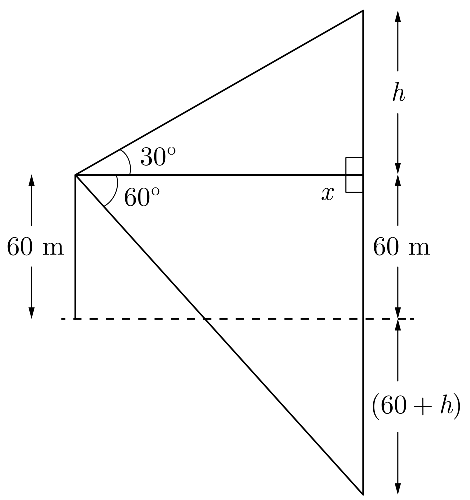
$$t = 3$$

Hence, car takes 3 seconds.

91. An angle of elevation of a cloud from a point 60 m above the surface of the water of a lake is 30° and the angle of depression of its shadow in water is 60°. Find the height of the cloud from the surface of water.

Ans : [Board Term-2 Delhi 2017]

As per given in question we have drawn figure below.



Here $\frac{h}{x} = \tan 30^\circ = \frac{1}{\sqrt{3}}$

$$x = h\sqrt{3} \quad ...(1)$$

and $\frac{h+60+60}{x} = \tan 60^\circ$

$$\frac{h+120}{x} = \sqrt{3}$$

$$h+120 = x\sqrt{3} \quad ...(2)$$

From (1) and (2) we get

$$h+120 = \sqrt{3}\,h \times \sqrt{3}$$

$$h+120 = 3h$$

$$h = \frac{120}{2} = 60 \text{ m}$$

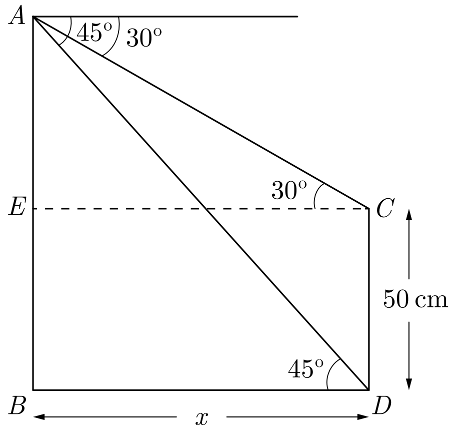
Hence height of cloud from surface of water

$$= 60+60 = 120 \text{ m}$$

92. The angle of depression of the top and bottom of a building 50 metres high as observed from the top of a tower are 30° and 45° respectively. Find the height of the tower and also the horizontal distance between the building and the tower.

Ans : [Board Term-2 SQP 2018]

Let CD be the building of height 50 m and AB be the tower of height h. Angle of depressions of top and bottom are given 30° and 60° respectively. As per given in question we have drawn figure below.



Let distance between BD be x.

Now, in right ΔABD we have

$$\frac{AB}{BD} = \tan 45^\circ$$

$$\frac{h}{x} = 1 \Rightarrow h = x \quad ...(1)$$

In right ΔAEC we have

, $$\frac{AE}{EC} = \tan 30^\circ$$

$$\frac{h-50}{x} = \frac{1}{\sqrt{3}}$$

$$x = h\sqrt{3} - 50\sqrt{3} \quad ...(2)$$

From (1) and (2) we get

$$h = h\sqrt{3} - 50\sqrt{3}$$

$$h\sqrt{3} - h = 50\sqrt{3}$$

$$h(\sqrt{3}-1) = 50\sqrt{3}$$

$$h = \frac{50\sqrt{3}}{\sqrt{3}-1} = \frac{50\sqrt{3}(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$$

$$= \frac{50(3+\sqrt{3})}{3-1}$$

$$h = 25(3+\sqrt{3})$$

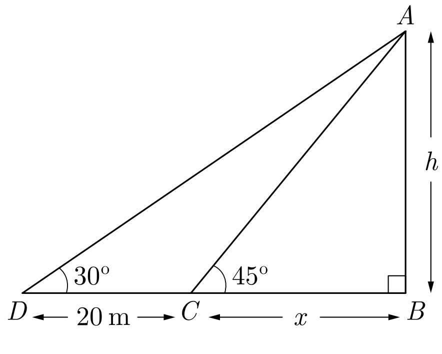
$$= 25 \times 4.732 = 118.3 \text{ m}$$

Hence, the height of tower = distance between building and tower = 118.3 m

93. An observer finds the angle of elevation of the top of the tower from a certain point on the ground as 30°. If the observer moves 20 m, towards the base of the tower, the angle of elevation of the top increase by 15°, find the height of the tower.

Ans : [Board Term-2 Delhi 2017]

Let AB be the tower of height h. Angle of elevation from point D and C are given 30° and 45° respectively. As per given in question we have drawn figure below.



Here $CB = x$ and $DC = 20$ m

Now in right $\Delta\, ABC$,

$$\frac{AB}{BC} = \tan 45^\circ$$

$$\frac{h}{x} = 1$$

$$h = x$$

In right $\Delta\, ABD$ we have

$$\frac{AB}{DB} = \tan 30^\circ$$

$$\frac{h}{(20+x)} = \frac{1}{\sqrt{3}}$$

$$h\sqrt{3} = 20 + x$$

Substituting the value of x from (1) in (2)

$$h\sqrt{3} = 20 + h$$

$$h\sqrt{3} - h = 20$$

$$h(\sqrt{3}-1) = 20$$

$$h = \frac{20}{\sqrt{3}-1} = \frac{20(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$$

$$= \frac{20(\sqrt{3}+1)}{3-1}$$

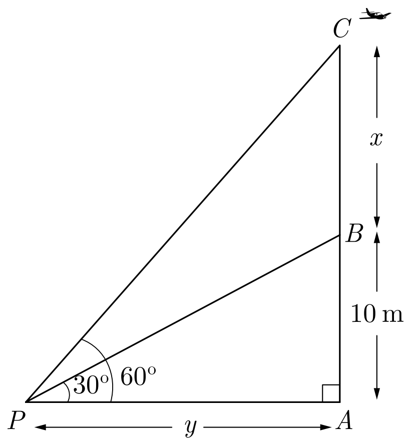
$$= 10(\sqrt{3}+1)$$

Hence, the height of tower $= 10(\sqrt{3}+1)$ m

94. From a point P on the ground, the angles of elevation of the top of a 10 m tall building and a helicopter, hovering at some height vertically over the top of the building are 30° and 60° respectively. Find the height of the helicopter above the ground.

Ans : [Board Term-2 OD Compt. 2017]

Let AB be the building of height 10 m and the height of the helicopter from top the building be x. As per given in question we have drawn figure below.



Let the distance between point and building be y.
Height of the helicopter from ground

$$= (10 + x) \text{ m}$$

In right $\Delta\, BAP$ we have

$$\frac{AB}{BP} = \tan 30^\circ$$

$$\frac{10}{y} = \frac{1}{\sqrt{3}}$$

$$y = 10\sqrt{3} \qquad ...(1)$$

In right $\Delta\, CAP$,

$$\frac{AC}{PA} = \tan 60^\circ$$

$$\frac{10+x}{y} = \sqrt{3}$$

$$10 + x = y\sqrt{3} \qquad ...(2)$$

From (1) and (2) we have

$$10 + x = 10\sqrt{3} \times \sqrt{3} = 30$$

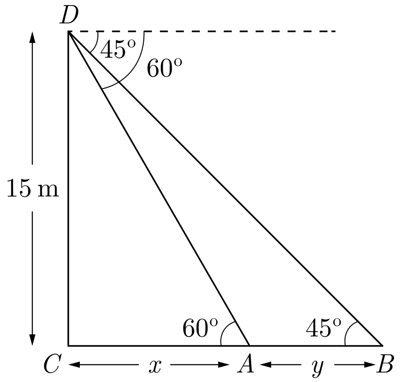
$$x = 20$$

Hence height of the helicopter is 20 m.

95. Two points A and B are on the same side of a tower and in the same straight line with its base. The angle of depression of these points from the top of the tower are 60° and 45° respectively. If the height of the tower is 15 m, then find the distance between these points.

Ans : [Board Term-2 OD 2017]

Let CD be the tower of height 15 m. Let A and B point on same side of tower As per given in question we have drawn figure below.



In right $\Delta\, DCA$ we have

$$\frac{DC}{CA} = \tan 60^\circ$$

$$\frac{15}{x} = \sqrt{3}$$

$$x = \frac{15}{\sqrt{3}} = 5\sqrt{3}$$

In right $\Delta\, DCB$ we have

$$\frac{DC}{CB} = \tan 45^\circ$$

$$\frac{15}{x+y} = 1$$

$$x + y = 15$$

$$5\sqrt{3} + y = 15$$

$$y = 15 - 5\sqrt{3}$$

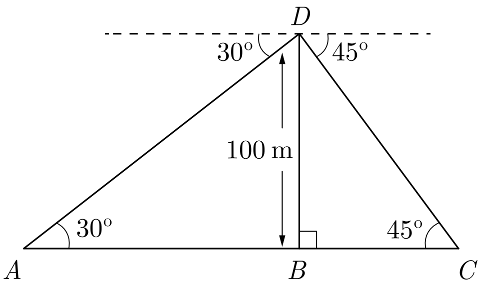
$$= 5(3 - \sqrt{3}) \text{ m}$$

Hence, the distance between points $= 5\ 3 - \sqrt{3}$ m

96. From the top of a tower, 100 m high, a man observes two cars on the opposite sides of the tower and in same straight line with its base, with angles of depression 30° and 45°. Find the distance between the cars. [Take $\sqrt{3} = 1.732$]

Ans : [Board Term-2 OD Compt. 2017]

Let BD be the tower of height 100 m. Let A and C be location of car on opposite side of tower. As per given in question we have drawn figure below.



In right $\Delta\, ABD$,

$$\angle DAB = 30^\circ$$

In $\Delta\, BDC$, $\angle BCD = 45^\circ$

also, $BD = 100$ m

In right ΔABD we have,

$$\tan 30^\circ = \frac{DB}{AB}$$

$$\frac{1}{\sqrt{3}} = \frac{100}{AB}$$

$$AB = 100\sqrt{3} \text{ m}$$

In right ΔDBC we have,

$$\tan 45^\circ = \frac{DB}{BC}$$

$$1 = \frac{100}{BC}$$

$$BC = 100 \text{ m}$$

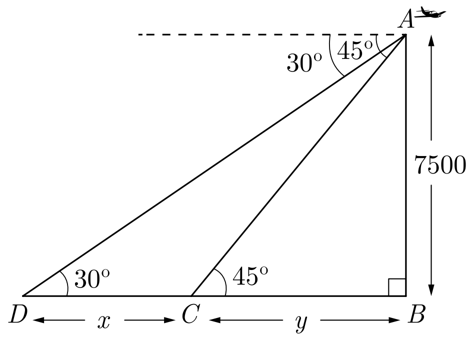
Now, $AB + BC = 100 + 100\sqrt{3} = 100(\sqrt{3} + 1)$

$$= 100 + 173.2 = 273.2 \text{ m}$$

97. The angle of depression of two ships from an aeroplane flying at the height of 7500 m are 30° and 45°. if both the ships are in the same that one ship is exactly behind the other, find the distance between the ships.

Ans : [Board Term-2 Foreign 2017]

Let A, C and D be the position of aeroplane and two ship respectively. Aeroplane is flying at 7500 m height from point B. As per given in question we have drawn figure below.



In right ΔABC we have

$$\frac{AB}{BC} = \tan 45^\circ$$

$$\frac{7500}{y} = 1$$

$$y = 7500 \qquad ...(1)$$

In right ΔABD we have

$$\frac{AB}{BD} = \tan 30^\circ$$

$$\frac{7500}{x+y} = \frac{1}{\sqrt{3}}$$

$$x + y = 7500\sqrt{3} \qquad ...(2)$$

Substituting the value of y from (1) in (2) we have

$$x + 7500 = 7500\sqrt{3}$$

$$x = 7500\sqrt{3} - 7500$$

$$= 7500(\sqrt{3} - 1)$$

$$= 7500(1.73 - 1)$$

$$= 7500 \times 0.73$$

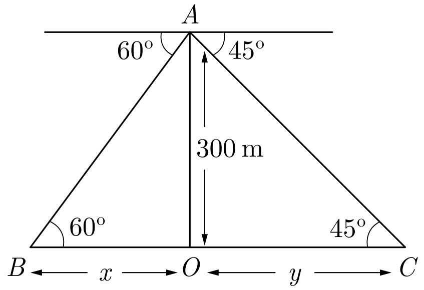
$$= 5475 \text{ m}$$

Hence, the distance between two ships is 5475 m.

98. An aeroplane is flying at a height of 300 m above the ground. Flying at this height the angle of depression from the aeroplane of two points on both banks of a respectively. Find the width of the river. River in opposite direction are 45° and 60°.

Ans : [Board Term-2 OD 2017]

Let A be helicopter flying at a height of 300 m above the point O on ground. Let B and C be the bank of river. As per given in question we have drawn figure below.



Let BO be x and OC be y.

In right ΔAOC we have

$$\frac{AO}{OC} = \tan 45^\circ$$

$$\frac{300}{y} = 1 \Rightarrow y = 300$$

In right ΔAOB we have

$$\frac{AO}{BO} = \tan 60^\circ$$

$$\frac{300}{x} = \sqrt{3}$$

$$x\sqrt{3} = 300 \Rightarrow x = \frac{300}{\sqrt{3}} = 100\sqrt{3}$$

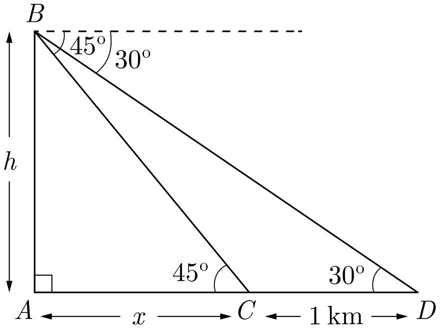
$$BC = y + x = 300 + 100\sqrt{3}$$
$$= 300 + 100 \times 1.732 = 473.2 \text{ m}$$

Hence, the width of river is 473.2 m.

99. From the top of a hill, the angle of depression of two consecutive kilometre stones due east are found to be 45° and 30° respectively. Find the height of the hill. [Use $\sqrt{3} = 1.73$]

Ans : [Board Term-2 OD 2016]

Let AB be the hill of height h. Angle of depression from point D and C are given 30° and 45° respectively. As per given in question we have drawn figure below.



In right $\Delta\, ABC$ we have

$$\frac{AB}{AC} = \tan 45^\circ$$

$$\frac{h}{x} = 1 \Rightarrow h = x$$

In right $\Delta\, ABD$ we have

$$\frac{AB}{AC + CD} = \tan 30^\circ$$

$$\frac{h}{x + 1000} = \frac{1}{\sqrt{3}}$$

$$h\sqrt{3} = h + 1000$$

$$h(\sqrt{3} - 1) = 1000$$

$$h = \frac{1000}{\sqrt{3} - 1} = \frac{1000(\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)}$$

$$= \frac{1000(\sqrt{3} + 1)}{3 - 1}$$

$$= 500(\sqrt{3} + 1) = 500(1.73 + 1)$$

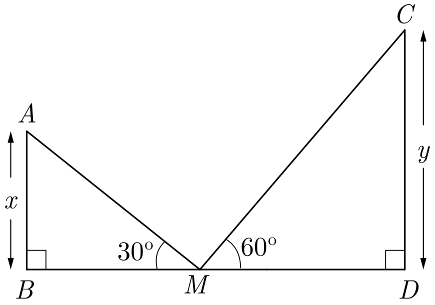
$$= 500 \times 2.73 = 1365$$

Hence height of the hill is 1365 m.

100. The tops of two towers of height x and y, standing on level ground, subtend angles of 30° and 60° respectively at the centre of the line joining their feet, then find $x : y$.

Ans : [Board Term-2 OD 2015]

Let AB be the tower of height x and CD be the tower of height y. Angle of depressions of both tower at centre point M are given 30° and 60° respectively. As per given in question we have drawn figure below.



Here M is the centre of the line joining their feet.

Let $BM = MD = z$

In right $\Delta\, ABM$ we have,

$$\frac{x}{z} = \tan 30^\circ$$

$$x = z \times \frac{1}{\sqrt{3}}$$

In right $\Delta\, CDM$ we have,

$$\frac{y}{z} = \tan 60^\circ$$

$$y = z \times \sqrt{3}$$

From (1) and (2), we get

$$\frac{x}{y} = \frac{z \times \frac{1}{\sqrt{3}}}{z \times \sqrt{3}}$$

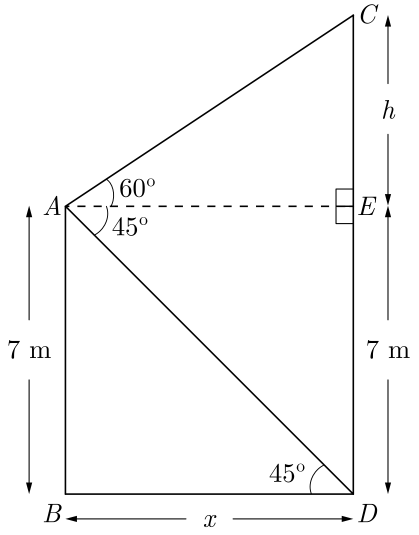
$$\frac{x}{y} = \frac{1}{3}$$

Thus $\quad x : y = 1 : 3$

101. From the top of a 7 m high building, the angle of elevation of the top of a tower is 60° and the angle of depression of its foot is 45°. Find the height of the tower. (Use $\sqrt{3} = 1.732$)

Ans : [Board Term-2 Foreign 2013]

Let AB be the building of height 7 m and CD be the tower of height h. Angle of depressions of top and bottom are given 30° and 60° respectively. As per given in question we have drawn figure below.



Here $\angle CBD = \angle ECB = 45^\circ$ due to alternate angles.

In right ΔABC we have

$$\frac{AB}{BC} = \tan 45^\circ$$

$$\frac{7}{x} = 1 \Rightarrow x = 7$$

In right ΔAEC we have

$$\frac{CE}{AE} = \tan 60^\circ$$

$$\frac{h-7}{x} = \sqrt{3}$$

$$h - 7 = x\sqrt{3} = 7\sqrt{3}$$

$$h = 7\sqrt{3} + 7$$

$$= 7(\sqrt{3}+1)$$

$$= 7(1.732+1)$$

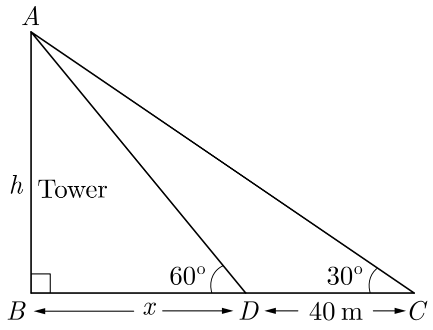
Hence, height of tower $= 19.124$ m

102. The shadow of a tower standing on a level ground is found to be 40 m longer when the Sun's altitude is 30°, then when it is 60°. Find the height of the tower.

Ans : [Board Term-2 OD 2011]

Let AB be the tower of height h. Let BC be the shadow at 60° and BD be shadow at 30°.

As per given in question we have drawn figure below.



In right ΔABC we get,

$$\tan 60^\circ = \frac{AB}{BC}$$

$$\sqrt{3} = \frac{h}{x} \Rightarrow h = \sqrt{3}\,x$$

In right ΔABD we have,

$$\tan 30^\circ = \frac{AB}{BC+40}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{x+40}$$

$$x + 40 = \sqrt{3}\,h = \sqrt{3} \times \sqrt{3}\,x = 3x$$

$$40 = 2x \Rightarrow x = 20 \text{ m}$$

$$h = \sqrt{3} \times 20 = 20\sqrt{3} \text{ m}$$

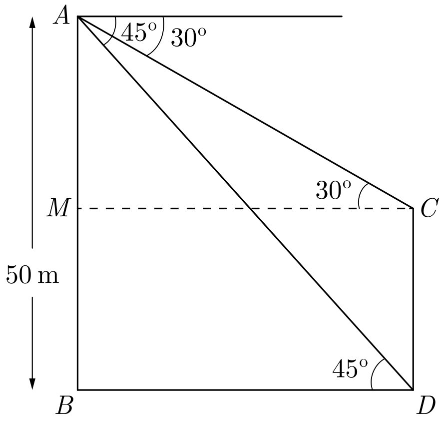
Thus height of tower is $20\sqrt{3}$ m.

103. From the top of a tower of height 50 m, the angles of depression of the top and bottom of a pole are 30°

and 45° respectively. Find :
(1) How far the pole is from the bottom of the tower,
(2) The height of the pole. (Use $\sqrt{3} = 1.732$)

Ans : [Board Term-2 Foreign 2015]

Let AB be the tower of height 50 m and CD be the pole of height h. From the top of a tower of height 50 m, the angles of depression of the top and bottom of a pole are 30° and 45° respectively. As per given in question we have drawn figure below.



In right $\Delta\,ABD$ we have,

$$\tan 45^\circ = \frac{AB}{BD} = 1$$

$$1 = \frac{50}{x} \Rightarrow x = 50 \text{ m}$$

(1) Thus distance of pole from bottom of tower is 50 m.

Now in $\Delta\,AMC$ we have

$$\tan 30^\circ = \frac{AM}{MC} = \frac{AM}{x}$$

$$AM = \frac{50}{\sqrt{3}} \text{ or } 28.87 \text{ m.}$$

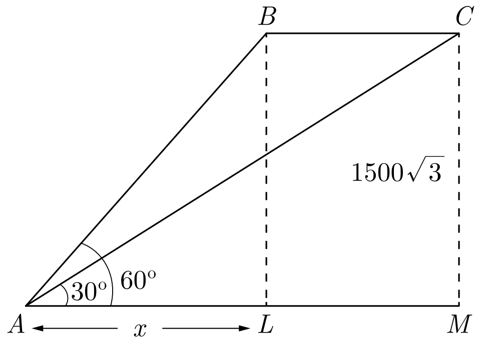
(2) Height pole $\quad h = CD = BM$

$$= 50 - 28.87 = 21.13 \text{ m.}$$

104. The angle of elevation of an aeroplane from a point A on the ground is 60°. After a flight of 15 seconds, the angle of elevation changed to 30°. If the aeroplane is flying at a constant height of $1500\sqrt{3}$ m, find the speed of the plane in km/hr.

Ans : [Board Term-2 OD 2015]

Let A be the point on ground, B and C be the point of location of aeroplane at height of $1500\sqrt{3}$ m. As per given in question we have drawn figure below.



In right $\Delta\,BAL$

$$\frac{BL}{AL} = \tan 60^\circ$$

$$\frac{1500\sqrt{3}}{x} = \sqrt{3} \qquad BL = CM = 1500\sqrt{3}$$

$$x = 1500 \text{ m.}$$

In right $\Delta\,CAM$ we have

$$\frac{CM}{AL + LM} = \tan 30^\circ$$

$$\frac{1500\sqrt{3}}{x + y} = \frac{1}{\sqrt{3}}$$

$$x + y = 1500 \times 3$$

$$1500 + y = 4500 \Rightarrow y = 3000 \text{ m.}$$

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} = \frac{y}{t}$$

$$= \frac{3000}{15} = 200 \text{ m/s}$$

$$= \frac{200}{1000} \times 60 \times 60$$

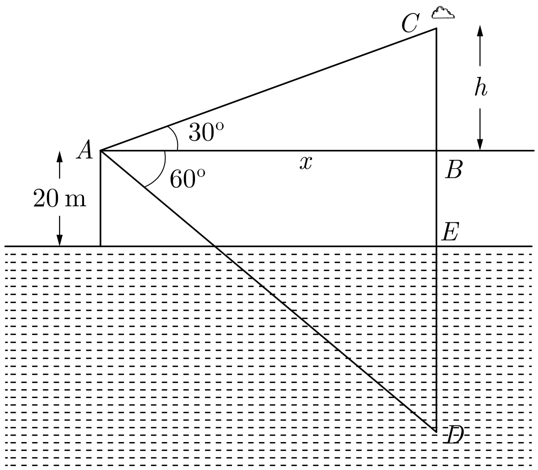
$$= 720 \text{ km/hr.}$$

Hence, the speed of the aeroplane is 720 km/hr.

105. At a point A, 20 metre above the level of water in a lake, the angle of elevation of a cloud is 30°. The angle of depression of the reflection of the cloud in the lake, at A is 60°. Find the distance of the cloud from A ?

Ans : [Board Term-2 OD 2015]

As per given in question we have drawn figure below. Here cloud is at C, D is reflection of cloud in water.



In right ΔABC we have

$$\frac{h}{x} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$x = \sqrt{3}\,h \qquad ...(1)$$

Here $DE = EC$ because D is reflection of cloud and E is at water level.

In right ΔABD we have

$$\frac{BD}{BA} = \tan 60^\circ$$

$$\frac{DE + EB}{x} = \sqrt{3}$$

$$\frac{EC + EB}{x} = \sqrt{3}$$

$$\frac{h + 20 + 20}{x} = \sqrt{3}$$

$$h + 40 = \sqrt{3}\,x \qquad ...(2)$$

From (1) and (2),

$$h + 40 = \sqrt{3} \times \sqrt{3}\,h = 3h$$

$$h = 20 \text{ m}$$

$$x = \sqrt{3}\,h = 20\sqrt{3}$$

Now $$AC = \sqrt{h^2 + x^2}$$

$$= \sqrt{(20)^2 + (20\sqrt{3})^2}$$

$$= \sqrt{400 + 1200}$$

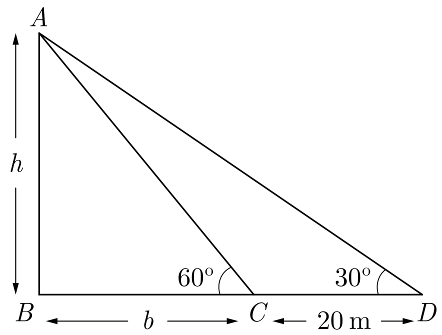
$$= 40 \text{ m.}$$

Hence distance of the cloud is 40 m.

106. A person standing on the bank of a river, observes that the angle of elevation of the top of the tree standing on the opposite bank is 60°. When he retreats 20 m from the bank, he finds the angle of elevation to be 30°. Find the height of the tree and the breadth of the river.

Ans : [Board Term-2 OD 2012]

Let AB be the tree of height h and breadth of river be b. As per given in question we have drawn figure below. Here point C and D are the location of person .



In right ΔABC we have,

$$\frac{h}{b} = \tan 60^\circ = \sqrt{3}$$

$$h = \sqrt{3}\,b \qquad ...(1)$$

In right ΔABD we have

$$\frac{h}{b + 20} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$h = \frac{b + 20}{\sqrt{3}} \qquad ...(2)$$

From (1) and (2) we have

$$b\sqrt{3} = \frac{b + 20}{\sqrt{3}}$$

$$3b = b + 20 \Rightarrow b = 10 \text{ m}$$

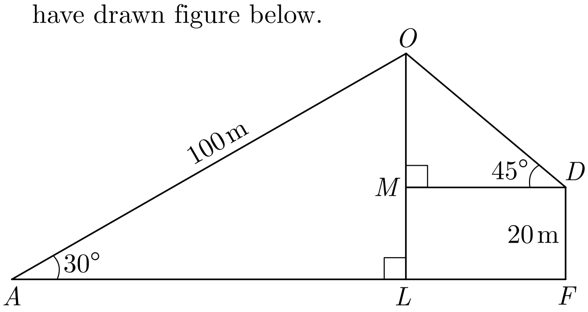
$$h = b\sqrt{3} = 10 \times 1.73 = 17.3 \text{ m}$$

Thus height of tree is 17.3 m and breadth of river is 10 m.

107. A boy observes that the angle of elevation of a bird flying at a distance of 100 m is 30°. At the same distance from the boy, a girl finds the angle of elevation of the same bird from a building 20 m high is 45°. Find the distance of the bird from the girl.

Ans : [Board Term-2 OD 2014

Let O be the position of the bird and B be the position of the boy. Let FG be the building and G be the position of the girl. As per given in question we have drawn figure below.



In right $\Delta\, OLB$ we have

$$\frac{OL}{BO} = \sin 30^\circ$$

$$\frac{OL}{100} = \frac{1}{2} \Rightarrow OL = 50 \text{ m}$$

$$OM = OL - ML$$

$$= OL - FG = 50 - 20 = 30 \text{ m}$$

In right $\Delta\, OMG$ we have

$$\frac{OM}{OG} = \sin 45^\circ$$

$$\frac{OM}{OG} = \frac{1}{\sqrt{2}}$$

$$\frac{30}{OG} = \frac{1}{\sqrt{2}}$$

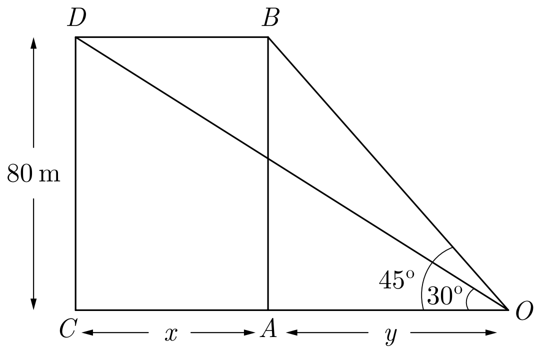
$$OG = 30\sqrt{2} \text{ m}$$

Hence, distance of the bird from the girl is $30\sqrt{2}$ m.

108. A bird sitting on the top of a 80 m high tree. From a point on the ground, the angle of elevation of the bird is 45°. The bird flies away horizontally in such a way that it remained at a constant height from the ground. After 2 seconds, the angle of elevation of the bird from the same point is 30°. Find the speed of flying of the bird. (Take $\sqrt{3} = 1.732$)

Ans : [Board Term-2 Delhi 2016]

Let CD be the tree of height 80 m and bird is sitting at D. Point O on ground is reference point from where we observe bird. As per given in question we have drawn figure below.



In right AOB we have

$$\tan 45^\circ = \frac{80}{y}$$

$$y = 80$$

In right DOC we have

$$\tan 30^\circ = \frac{80}{x+y}$$

$$\frac{1}{\sqrt{3}} = \frac{80}{x+y}$$

$$x+y = 80\sqrt{3}$$

$$x = 80\sqrt{3} - y = 80\sqrt{3} - 80$$

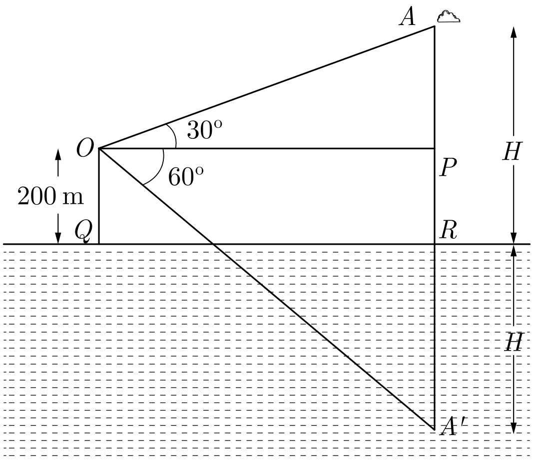
$$= 80(\sqrt{3} - 1) = 58.4 \text{ m.}$$

Hence, speed of bird $= \frac{58.4}{2} = 29.2$ m

109. The angle of elevation of a cloud from a point 200 m above the lake is 30° and the angle of depression of its reflection in the lake is 60°, find the height of the cloud above the lake.

Ans : [Board Term-2 OD 2012, 2011]

Let H be the height of cloud at A from lake. As per given in question we have drawn figure below.



Here A is cloud and A' is refection of cloud.

In right $\Delta\, AOP$ we have

$$\tan 30^\circ = \frac{PA}{OP}$$

$$\frac{1}{\sqrt{3}} = \frac{H-200}{OP}$$

$$OP = (H-120)\sqrt{3} \qquad ...(1)$$

In right $\Delta\, OPA'$ we have

$$\tan 60^\circ = \frac{PA'}{OP}$$

$$\sqrt{3} = \frac{H+200}{OP}$$

$$OP = \frac{H+200}{\sqrt{3}} \qquad ...(2)$$

From (1) and (2), we get

$$\frac{H+200}{\sqrt{3}} = \sqrt{3}(H-200)$$

$$H+200 = 3(H-200)$$

$$H+200 = 3H-600$$

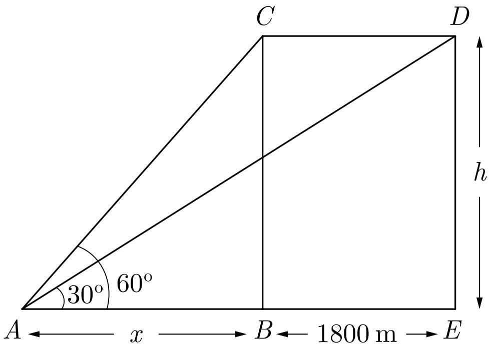
$$2H = 800 \Rightarrow H = 400$$

Thus height of cloud is 400 m.

110. The angle of elevation of a jet fighter point A on ground is 60°. After flying 10 seconds, the angle changes to 30°. If the jet is flying at a speed of 648 km/hour, find the constant height at which the jet is flying.

Ans : [Board Term-2 Delhi 2012]

Let C and D are the point of location of jet at height h. Point B and E are foot print on ground of get at thee location. As per given in question we have drawn figure below.



In 3600 sec distance travelled by plane = 648000 m

In 10 sec distance travelled by plane $= \frac{648000}{3600} \times 10$

$$= 1800 \text{ m}$$

In right $\Delta\, ABC$, we have

$$\frac{h}{x} = \tan 60^\circ = \sqrt{3}$$

$$h = x\sqrt{3} \qquad ...(1)$$

In right $\Delta\, ADE$ we have

$$\frac{h}{x+1800} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$h = \frac{x+1800}{\sqrt{3}} \qquad ...(2)$$

From equations (1) and (2), we get

$$x\sqrt{3} = \frac{x+1800}{\sqrt{3}}$$

$$3x = x+1800$$

$$2x = 1800$$

$$x = 900 \text{ m}$$

$$h = x\sqrt{3}$$

$$= 900 \times 1.732$$

$$= 1558.5 \text{ m}$$

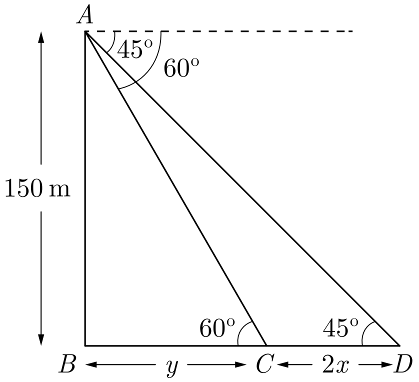
Thus height of jet is 1558.8 m.

111. A moving boat observed from the top of a 150 m high cliff moving away from the cliff. The angle of depression of the boat changes from 60° to 45° in 2 minutes. Find the speed of the boat.

Ans : [Board Term-2 Delhi 2017]

Let AB be the cliff of height 150 m. Let C and D be the point of boat at 60° and 45°. Let the speed of the boat be x m/min. Let BC be y

As per given in question we have drawn figure below.



Here distance covered in 2 minutes is $2x$.

Thus $CD = 2x$

In right ΔABD we have

$$\frac{AB}{BC} = \tan 60^\circ$$

$$\frac{150}{y} = \sqrt{3}$$

$$y = \frac{150}{\sqrt{3}} = 50\sqrt{3} \qquad ...(1)$$

In right ΔABD we have

$$\frac{AB}{BD} = \tan 45^\circ$$

$$\frac{150}{y+2x} = 1$$

$$y + 2x = 150 \qquad ...(2)$$

From equations (1) and (2), we get

$$50\sqrt{3} + 2x = 150$$

$$2x = 150 - 50\sqrt{3}$$

$$2x = 50(3-\sqrt{3})$$

$$x = 25(3-\sqrt{3})$$

$$\text{Speed of the boat} = 25(3-\sqrt{3}) \text{ m/min.}$$

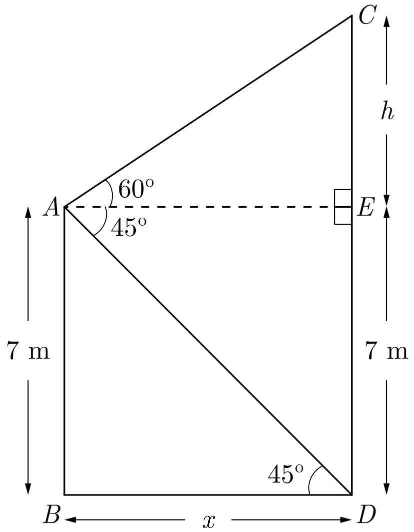
$$= \frac{25(3-\sqrt{3})\times 60}{1000}$$

$$= \frac{3}{2}(3-\sqrt{3}) \text{ km/hr.}$$

112. From the top of a 7 m high building the angle of elevation of the top of a tower is 60° and the angle of depression of its foot is 45°. Find the height of the tower.

Ans : [Board Term-2 Delhi 2017]

Let AB be the building of height 7 m and CD be the tower. Let distance between two be x. Angle of depressions of top and bottom of tower are given 60° and 45° respectively. As per given in question we have drawn figure below.



CD be the height of tower $= (7+h)$ m

$$BD = AE = x \text{ m}$$

In right ΔABD we have

$$\frac{AB}{BD} = \tan 45^\circ$$

$$\frac{7}{x} = 1 \Rightarrow x = 7 \text{ m} \qquad ...(1)$$

In right ΔCEA we have

$$\frac{CE}{AE} = \tan 60^\circ$$

$$\frac{h-7}{x} = \sqrt{3}$$

$$h - 7 = x\sqrt{3} \qquad ...(2)$$

Substituting values of x we have

$$h - 7 = 7\sqrt{3}$$

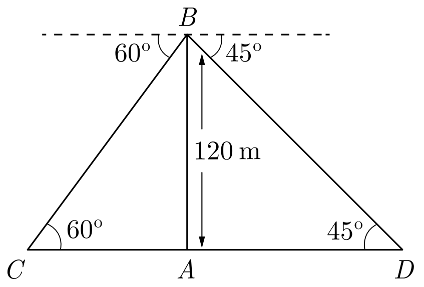
$$h = 7 + 7\sqrt{3} = 7(1+\sqrt{3}) \text{ m}$$

Hence, the height of tower is $7(1+\sqrt{3})$ m

113. From the top of a 120 m high tower, a man observes two cars on the opposite sides of the tower and in straight line with the base of tower with angles of repression as 60° and 45°. Find the distance between two cars.

Ans : [Delhi Compt. 2017]

Let AB be the tower of height 120 m. Let C and D be location of car on opposite side of tower. As per given in question we have drawn figure below.



In right $\Delta\, BAD$ we have

$$\frac{AB}{AD} = \tan 45^\circ$$

$$\frac{120}{AB} = 1$$

$$AB = 120$$

In right $\Delta\, BAC$ we have

$$\frac{AB}{CA} = \tan 60^\circ$$

$$\frac{120}{CA} = \sqrt{3}$$

$$CA = \frac{120}{\sqrt{3}} = 40\sqrt{3}$$

$$\begin{aligned} CD &= CA + AD \\ &= 120 + 40\sqrt{3} \\ &= 120 + 40 \times 1.732 \\ &= 189.28 \text{ m} \end{aligned}$$

Hence the distance between two men is 189.28 m.