SAMPLE QUESTION PAPER (STANDARD) - 02

Class 10 - Mathematics

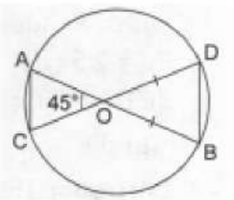
Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section A has 20 MCQs carrying 1 mark each.
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with subparts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E.
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

Section A

1. In the given figure, O is the point of intersection of two chords AB and CD such that OB = OD and ∠AOC = 45°. Then, △OAC and △ODB are **[1]**



a) equilateral and similar b) equilateral but not similar

c) isosceles but not similar d) isosceles and similar

2. If the zeroes of a quadratic polynomial $ax^2 + bx + c$, $c \neq 0$ are equal, then **[1]**

a) c and a have opposite sign b) b and c have opposite sign

c) c and a have the same sign d) b and c have the same sign

3. I am three times as old as my son. Five years later, I shall be two and a half times as old as my son. My present age is **[1]**

a) 20 years b) 45 years

c) 15 years d) 50 years

4. If x = -y and y > 0, which of the following is wrong? **[1]**

a) xy < 0 b) x + y = 0

c) $\frac{1}{x} - \frac{1}{y} = 0$

d) $x^2y > 0$

5. In $\triangle$ABC and $\triangle$DEF, it is given that $\frac{AB}{DE} = \frac{BC}{FD}$ then **[1]**

a) ∠A = ∠D

b) ∠B = ∠D

c) ∠B = ∠E

d) ∠A = ∠F

6. A number is selected from first 50 natural numbers. What is the probability that it is a multiple of 3 or 5? **[1]**

a) $\frac{21}{50}$

b) $\frac{12}{25}$

c) $\frac{23}{50}$

d) $\frac{13}{25}$

7. If sinα = $\frac{1}{\sqrt{2}}$ and tanβ = 1, then the value of $\cos(\alpha + \beta)$ is **[1]**

a) 3

b) 1

c) 2

d) 0

8. In the formula $\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i}$ for finding the mean of grouped data d_i's are deviations from a of **[1]**

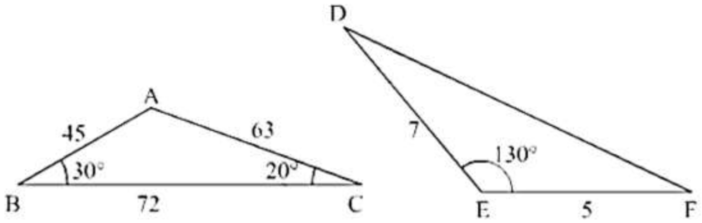
a) upper limits of the classes

b) lower limits of the classes

c) mid points of the classes

d) frequencies of the class marks

9. In the given figures the measures of ∠D and ∠F are respectively **[1]**



a) $20^o, 30^o$.

b) $30^o, 20^o$.

c) $50^o, 40^o$.

d) $40^o, 50^o$.

10. If $a = 2^3 \times 3, b = 2 \times 3 \times 5, c = 3^n \times 5$ and LCM (a, b, c) $= 2^3 \times 3^2 \times 5$, then n = **[1]**

a) 1

b) 4

c) 3

d) 2

11. A cyclist takes 2 hours less to cover a distance of 200 km, if he increases his speed by 5 km/hr. Then his original speed is **[1]**

a) 26 km/hr

b) 20 km/hr

c) 24 km/hr

d) 25 km/hr

12. The distance between the points A (0, 6) and B (0, –2) is **[1]**

a) 8

b) 4

c) 6

d) 2

13. In the following distribution : **[1]**

Monthly income	**Number of families**
More than 10000	100
More than 13000	85

More than 16000	69
More than 19000	50
More than 22000	33
More than 25000	15

the number of families having income range (in ₹) 16000 – 19000 is

a) 15

b) 17

c) 16

d) 19

14. If 7 tan θ = 4 then $\frac{(7\sin\theta-3\cos\theta)}{(7\sin\theta+3\cos\theta)}=?$ **[1]**

a) $\frac{3}{7}$

b) $\frac{1}{7}$

c) $\frac{5}{14}$

d) $\frac{5}{7}$

15. The length of the shadow of a 20 m tall pole on the ground when the sun's elevation is 45° is **[1]**

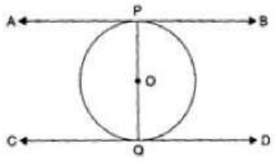
a) 20 m

b) $20\sqrt{2}$ m

c) 40 m

d) $20\sqrt{3}$ m

16. The distance between two parallel tangents of a circle of radius 3 cm is **[1]**



a) 6 cm

b) 3 cm

c) 4.5 cm

d) 12 cm

17. In triangle ABC, D, E and F are the midpoints of sides BC, CA and AB respectively and $\frac{AB}{DE}=\frac{BC}{FE}=\frac{CA}{FD}, then$ **[1]**

a) $\Delta BCA \sim \Delta FDE$

b) $\Delta CBA \sim \Delta FDE$

c) $\Delta FDE \sim \Delta ABC$

d) $\Delta FDE \sim \Delta CAB$

18. The quadratic equation $ax^2 + 2x + a = 0$ has two distinct roots, if **[1]**

a) a = $\pm$ 1

b) a = 0

c) a = 0, 1

d) 0 = - 1, 0

19. **Assertion (A):** Product of zeroes of polynomial $x^3 - 3x^2 + 7x - 10$ is 10. **[1]**

Reason (R): Product of zeroes of polynomial: $ax^2 + bx + c$ is given by $\frac{c}{a}$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** Two identical solid cubes of side 5 cm are joined end to end. The total surface area of the resulting cuboid is 350 cm^2. **[1]**

Reason (R): Total surface area of a cuboid is 2(lb + bh + hl)

a) Both A and R are true and R is the correct

b) Both A and R are true but R is not the explanation of A. | correct explanation of A.

c) A is true but R is false. | d) A is false but R is true.

Section B

21. Solve the quadratic equation by factorization: **[2]**

$9x^2 - 6b^2x - (a^4 - b^4) = 0$

22. If A(4, 3), B (-1, y), and C(3, 4) are the vertices of a right triangle ABC, right angled at A, then find the value of y. **[2]**

23. An army contingent of 612 members is to march behind an army band of 48 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march? **[2]**

24. If m sinA + n cosA = p and m cosA - n sinA = q, prove that $m^2 + n^2 = p^2 + q^2$. **[2]**

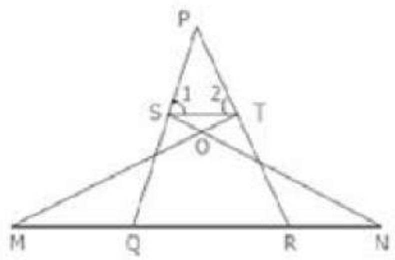
OR

If θ is a positive acute angle such that $\sec\theta = \cos ec 60^\circ$, find the value of $2\cos^2\theta - 1$.

25. Prove that a line draw through the mid point of one side of a triangle parallel to another side bisects the third side. **[2]**

OR

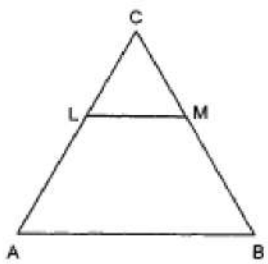
In the given figure, if $\angle 1 = \angle 2$ and $\triangle NSQ \cong \triangle MTR$ .Then prove that $\triangle PTS \cong \triangle PRQ$



Section C

26. Out of a group of swans, $\frac{7}{2}$ times the square root of the total number of swans are playing on the shore of a tank. Remaining two are playing, with amorous fight, in the water. What is the total number of swans? **[3]**

27. In Fig. $LM \parallel AB$. If AL = x - 3, AC = 2x, BM = x - 2 and BC = 2x + 3, find the value of x. **[3]**



28. The two opposite vertices of a square are (-1, 2) and (3, 2). Find the coordinates of the other two vertices. **[3]**

OR

Point A lies on the line segment PQ joining P(6, -6) and Q (-4, -1) in such a way that $\frac{PA}{PQ} = \frac{2}{5}$. If the point A also lies on the line 3x + k(y + 1)=0, find the value of k.

29. Show that $\frac{\sqrt{2}}{3}$ is irrational. **[3]**

30. The angle of elevation of the top of a tower at a point on the level ground is 30^o. After walking a distance of 100 m towards the foot of the tower along the horizontal line through the foot of the tower on the same level ground the angle of elevation to the top of the tower is 60^o, find the height of the tower. **[3]**

OR

A tower subtends an angle α at a point A in the plane of its base and the angle of depression of the foot of the tower at a point B which is at 'b' meters above A is β.

Prove that the height of the tower is b tan α cot β.

31. 100 surnames were randomly picked up from a local telephone directory and the frequency distribution of the number of letters in the English alphabets in the surnames was obtained as follows: **[3]**

Number of letters	1-4	4-7	7-10	10-13	13-16	16-19
Number of surnames	6	30	40	16	4	4

Determine the median number of letters in the surnames. Find the mean number of letters in the surnames? Also, find the modal size of the surnames.

Section D

32. The sum of the numerator and denominator of a fraction is 8. If 3 is added to both the numerator and denominator the fraction becomes $\frac{3}{4}$. Find the fraction. **[5]**

OR

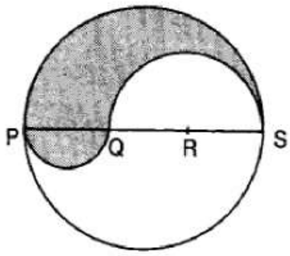
Abdul travelled 300 km by train and 200 km by taxi taking 5 hours 30 minutes. But, if he travels 260 km by train and 240 km by taxi, he takes 6 minutes longer. Find the speed of the train and that of the taxi.

33. In a right triangle ABC in which $\angle B = 90°$, a circle is drawn with AB as diameter intersecting the hypotenuse AC at P. Prove that the tangent to the circle at P bisects BC. **[5]**

34. Find the area of the segment of a circle of radius 12 cm whose corresponding sector central angle 60°. (Use π = 3.14). **[5]**

OR

PQRS is a diameter of a circle of radius 6 cm. The lengths PQ, QR and RS are equal. Semi-circles are drawn on PQ and QS as diameters as shown in Fig. Find the perimeter and area of the shaded region



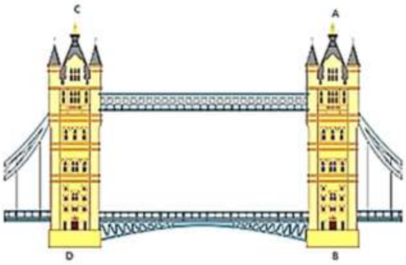
35. A number x is selected at random from the numbers 1,2,3 and 4. Another number y is selected at random from the numbers 1,4, 9 and 16. Find the probability that product of x and y is less than 16. **[5]**

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Tower Bridge is a Grade I listed combined bascule and suspension bridge in London, built between 1886 and 1894, designed by Horace Jones and engineered by John Wolfe Barry. The bridge is 800 feet (240 m) in length and consists of two bridge towers connected at the upper level by two horizontal walkways, and a central pair of bascules that can open to allow shipping.

In this bridge, two towers of equal heights are standing opposite each other on either side of the road, which is 80 m wide. During summer holidays, Neeta visited the tower bridge. She stood at some point on the road between these towers. From that point between the towers on the road, the angles of elevation of the top of the towers was 60^o and 30^o respectively.



(i) Find the distances of the point from the base of the towers where Neeta was standing while measuring the height.

(ii) Neeta used some applications of trigonometry she learned in her class to find the height of the towers without actually measuring them. What would be the height of the towers she would have calculated?

(iii) Find the distance between Neeta and top of tower AB?

OR

Find the distance between Neeta and top tower CD?

37. **Read the text carefully and answer the questions:** **[4]**

Sehaj Batra gets pocket money from his father every day. Out of pocket money, he saves money for poor people in his locality. On 1st day he saves ₹27.5 On each succeeding day he increases his saving by ₹2.5.

(i) Find the amount saved by Sehaj on 10^{th} day.

(ii) Find the amount saved by Sehaj on 25^{th} day.

OR

Find in how many days Sehaj saves ₹1400.

(iii) Find the total amount saved by Sehaj in 30 days.

38. **Read the text carefully and answer the questions:** **[4]**

Ashish is a Class IX student. His class teacher Mrs Verma arranged a historical trip to great Stupa of Sanchi. She explained that Stupa of Sanchi is great example of architecture in India. Its base part is cylindrical in shape. The dome of this stupa is hemispherical in shape, known as Anda. It also contains a cubical shape part called Hermika at the top. Path around Anda is known as Pradakshina Path.

(i) Find the volume of the Hermika, if the side of cubical part is 10 m.

(ii) Find the volume of cylindrical base part whose diameter and height 48 m and 14 m.

OR

If the diameter of the Anda is 42 m, then find the volume of the Anda.

(iii) If the volume of each brick used is 0.01 m^3, then find the number of bricks used to make the cylindrical base.

Solution

SAMPLE QUESTION PAPER (STANDARD) - 02

Class 10 - Mathematics

Section A

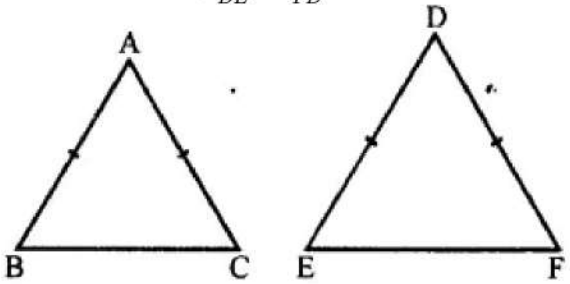
1. **(d)** isosceles and similar
   Explanation: In the given figure, O is the point of intersection of two chords AB and CD.
   OB = OD and $\angle$AOC = 45°
   $\angle$B = $\angle$D (Angles opposite to equal sides)
   $\angle$A = $\angle$D, $\angle$C = $\angle$B (Angles in the same segment)
   and $\angle$AOC = $\angle$BOD = 45° each
   $\triangle$OAC $\sim$ $\triangle$ODB (AAA axiom)
   OA = OC (Sides opposite to equal angles)
   $\triangle$OAC and $\triangle$ODB are isosceles and similar.

2. **(c)** c and a have the same sign
   Explanation: If the zeroes of a quadratic polynomial $ax^2 + bx + c, c \neq 0$ are equal, then $b^2 - 4ac = 0$. $\implies b^2 = 4ac$.
   Here b^2 is always positive
   And this is possible only if a and c are both positive or both negative. Hence both should have the same sign

3. **(b)** 45 years
   Explanation: Let my age and son's age be x and y years.
   Given, x = 3y
   x + 5 = $\frac{5(y+5)}{2}$
   $\Rightarrow$ 3y + 5 = $\frac{5(y+5)}{2}$
   $\Rightarrow$ 6y + 10 = 5y + 25
   $\Rightarrow$ y = 15
   x = 3 $\times$ 15 = 45
   Hence, my age and son's age are 45 years and 15 years.

4. **(c)** $\frac{1}{x} - \frac{1}{y} = 0$
   Explanation: Given that x = - y and y > 0
   $\frac{1}{x} - \frac{1}{y} = 0$
   $\Rightarrow \frac{1}{-y} - \frac{1}{y} = 0$
   $\Rightarrow \frac{-2}{y} \neq 0$
   Since y > 0, also $\frac{1}{y}$ > 0 but $\frac{-2}{y}$ <0
   Hence, it is not satisfied.

5. **(b)** $\angle$B = $\angle$D
   Explanation:
   In $\triangle$ABC and $\triangle$DEF, $\frac{AB}{DE} = \frac{BC}{FD}$



   For similarity,
   Here, included angles must be equal and these
   are $\angle$B = $\angle$D

6. **(c)** $\frac{23}{50}$
   Explanation: Total numbers = 1 to 50 = 50

Numbers which are multiples of 3 or 5, are 3,5, 6,9, 10, 12, 15, 18, 20, 21 , 24, 25, 27, 30, 33 35, 36, 39, 40, 42, 45, 48, 50 = 23

$\therefore P(E) = \frac{m}{n} = \frac{23}{50}$

7. **(d)** 0

Explanation: Given: sinα = $\frac{1}{\sqrt{2}}$

$\Rightarrow$ sinα = sin45^o

$\Rightarrow$ α = 45^o

And tanβ = 1

$\Rightarrow$ tanβ = tan45^o

$\Rightarrow$ β = 45^o

$\therefore$ cos(α + β) = cos(45^o + 45^o) = cos90^o = 0

8. **(c)** mid points of the classes

Explanation: We know that, $d_i = x_i - a$

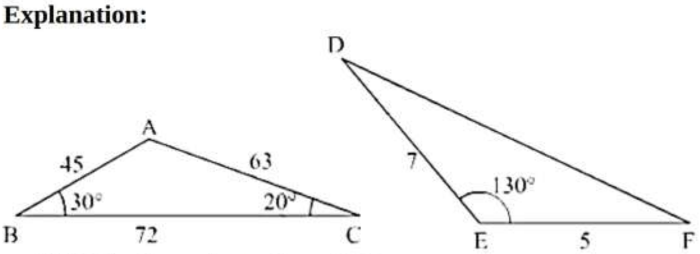
Where,

x_i are data or class mark and "a" is the assumed mean

i.e. d_i are the deviations of observations from assumed mean.

9. **(a)** 20^o, 30^o.

Explanation:



In ΔABC, $\angle$A + $\angle$B + $\angle$C = 180°

$\Rightarrow \angle A + 30^\circ + 20^\circ = 180^\circ$

$\Rightarrow A = 130^\circ$

Again, in ΔABC and ΔDEF,

$\frac{AB}{AC} = \frac{EF}{ED}$

$\angle A = \angle E = 130^\circ$

$\triangle ABC \sim \Delta EFD$ (SAS Similarity)

$\therefore \angle F = \angle B = 30^\circ$

$\angle D = \angle C = 20^\circ$

10. **(d)** 2

Explanation: LCM (a, b, c) $= 2^3 \times 3^2 \times 5$ .... (I)

we have to find the value of n

Also we have

$a = 2^3 \times 3$

$b = 2 \times 3 \times 5$

$c = 3^n \times 5$

We know that the while evaluating LCM, we take greater exponent of the prime numbers in the factorisation of the number.

Therefore, by applying this rule and taking $n \geq 1$ we get the LCM as

LCM (a, b, c) = $2^3 \times 3^n \times 5$ ..... (II)

On comparing (I) and (II) sides, we get:

$2^3 \times 3^2 \times 5 = 2^3 \times 3^n \times 5$

n = 2

11. **(b)** 20 km/hr

Explanation: Let the original speed be x km/h

$\therefore$ Time taken to cover 200 km at the rate of x km/h = $\frac{200}{x}$ hrs

New rate = (x + 5) km/h

$\therefore$ Time taken to cover 200 km at new rate = $\frac{200}{x+5}$ hrs

According to question, $\frac{200}{x} - \frac{200}{x+5} = 2$

$\Rightarrow 200\left[\frac{1}{x} - \frac{1}{x+5}\right] = 2$

$\Rightarrow 200\left[\frac{x+5-x}{x(x+5)}\right] = 2$

$\Rightarrow \frac{1000}{x^2+5x} = 2$

$\Rightarrow x^2 + 5x - 500 = 0$

$\Rightarrow x^2 + 25x - 20x - 500 = 0$

$\Rightarrow x(x + 25) -20(x + 25) = 0$

$\Rightarrow x(x + 25)(x - 20) = 0$

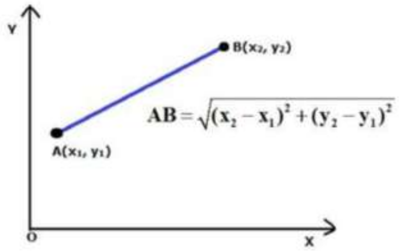
$\Rightarrow x + 25$ and $x - 20 = 0$

$\Rightarrow x = -25$ and $x = 20$

Therefore, the original speed is 20 km/h.

12. **(a)** 8

Explanation: By using the distance formula:



$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

Lets calculate the distance between the points (x_1, y_1) and (x_2, y_2)

We have;

$x_1 = 0, x_2 = 0$

$y_1 = 6, y_2 = - 2$

$d^2 = (0 - 0)^2 + (- 2 - 6)^2$

$d = \sqrt{(0)^2 + (-8)^2}$

$d = \sqrt{64}$

d = 8 units

So, the distance between A (0, 6) and B (0, - 2) = 8

13. **(d)** 19

Explanation: Between 16000 and 19000 we need to subtract the frequencies of these classes to get the desired result i.e., 69 - 50 = 19 .

14. **(b)** $\frac{1}{7}$

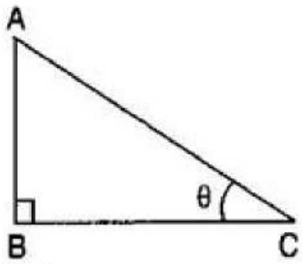
Explanation: Given, $\tan\theta = \frac{4}{7}$

$\therefore \quad \frac{(7\sin\theta - 3\cos\theta)}{(7\sin\theta + 3\cos\theta)} = \frac{(7\tan\theta - 3)}{(7\tan\theta + 3)}$ [Dividing numerator and denom. by $\cos\theta$]

$= \frac{\left(7\times\frac{4}{7} - 3\right)}{\left(7\times\frac{4}{7} + 3\right)} = \frac{(4-3)}{(4+3)} = \frac{1}{7}$

15. **(a)** 20 m

Explanation:



Given: Height of pole = AB = 20 m

And the angle of elevation $\theta = 45^\circ$

Let length of shadow of pole = BC = x meters

$\therefore \tan 45^\circ = \frac{AB}{BC}$

$\Rightarrow 1 = \frac{20}{x}$

$\Rightarrow x = 20$ m

Therefore, the length of the shadow of the pole is 20 m.

16. **(a)** 6 cm

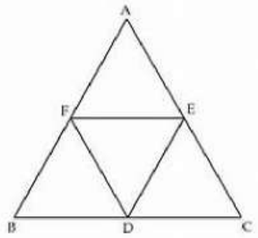
Explanation: Since the distance between two parallel tangents of a circle is equal to the diameter of the circle.

Given: Radius (OP) = 3 cm

$\therefore$ Diameter = 2 $\times$ Radius = 2 $\times$ 3 = 6 cm

17. **(d)** $\Delta FDE \sim \Delta CAB$

Explanation: Since $\frac{AB}{DE} = \frac{BC}{FE} = \frac{CA}{FD}$, $then$ as sides are in proportion, then by SSS similarity criteria, $\Delta FDE \sim \Delta CAB$



Also, FEDC is a parallelogram, then $\angle F = \angle C$

And, AFDF is a parallelogram, then $\angle D = \angle A$

And, BDEF is a parallelogram, then $\angle E = \angle B$

Therefore, by AAA similarity criteria $\Delta FDE \sim \Delta CAB$

18. **(a)** a = $\pm$ 1

Explanation: In the equation $ax^2 + 2x + a = 0$

$D = b^2 - 4ac = (2)^2 - 4 \times a \times a = 4 - 4a^2$

Roots are real and equal

D = 0

$\Rightarrow 4 - 4a^2 = 0$

$\Rightarrow 4 = 4a^2$

$\Rightarrow 1 = a^2$

$\Rightarrow a^2 = 1$

$\Rightarrow a^2 = (\pm 1)^2$

$\Rightarrow a = \pm 1$

19. **(b)** Both A and R are true but R is not the correct explanation of A.

Explanation: Product of zeroes of a cubic polynomial is given by $-\frac{d}{a}$.

20. **(d)** A is false but R is true.

Explanation: A is false but R is true.

Section B

21. We have,

$9x^2 - 6b^2x - (a^4 - b^4) = 0$

Here, Constant term = $a^4 - b^4 = (a^2 - b^2)(a^2 + b^2)$

Also, the coefficient of middle term is $- 6b^2 = [3(a^2 + b^2) - 3(a^2 - b^2)]$

Now using the above two values in given equation, $9x^2 - 6b^2x - (a^4 - b^4) = 0$

we have, $9x^2 - [3(a^2 + b^2) - 3(a^2 - b^2)]x - (a^2 - b^2)(a^2 + b^2) = 0$

$\Rightarrow 9x^2 - 3(a^2 + b^2)x + 3(a^2 - b^2)x - (a^2 - b^2)(a^2 + b^2) = 0$

$\Rightarrow 3x[3x - (a^2 + b^2)] + (a^2 - b^2)[3x - (a^2 + b^2)] = 0$

$\Rightarrow [3x + (a^2 - b^2)][3x - (a^2 + b^2)] = 0$

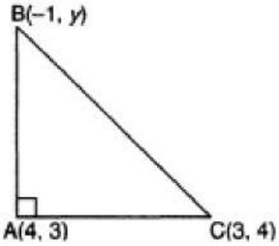
$\Rightarrow$either $[3x + (a^2 - b^2)] = 0$ or, $[3x - (a^2 + b^2)] = 0$

$\Rightarrow 3x = -(a^2 - b^2)$ or $3x = a^2 + b^2$

$\Rightarrow x=-\left(\frac{a^2-b^2}{3}\right)$ or $x=\frac{a^2+b^2}{3}$

$\Rightarrow x=\frac{b^2-a^2}{3}$ or $x=\frac{a^2+b^2}{3}$

Hence, the roots of given quadratic equation are $\frac{b^2-a^2}{3}$ and $\frac{a^2+b^2}{3}$

22. 

If A(4, 3), B (-1, y), and C(3,4) are the vertices of a right triangle ABC, right angled at A, then, we have to find the value of y.

By Pythagoras theorm,

$AB^2 + AC^2 = BC^2$

or, $(4 + 1)^2 + (3 - y)^2 + (3 - 4)^2 + (4 - 3)^2 = (3 + 1)^2 + (4 - y)^2$

or, $(5)^2 + (3 - y)^2 + (-1)^2 + (1)^2 = (4)^2 + (4 - y)^2$

or, $25 + 9 - 6y + y^2 + 1 + 1 = 16 + 16 - 8y + y^2$

or, $36 + 2y - 32 = 0$

or, $y = -2$

23. Clearly, the maximum number of columns = HCF (612, 48).

2	612
2	306
3	153
3	51
	17

2	48
2	24
2	12
2	6
	3

Now, $612 = 2\times 2\times 3\times 3\times 17=\left(2^2\times 3^2\times 17\right)$

and $48=2\times 2\times 2\times 2\times 3=\left(2^4\times 3\right)$.

$\therefore$ HCF $(612, 48)=\left(2^2\times 3\right)=(4\times 3)$

$\therefore$ HCF (612, 48)= 12.

$\therefore$ Maximum number of columns in which they can march = 12.

24. Given

m sinA + n cosA = p......(1)

m cosA - n sinA = q........(2)

Squaring (1) and (2) we get,

$m^2 \sin^2 A + n^2 \cos^2 A + 2mn \sin A \cos A = p^2$ ......(3)

$m^2 \cos^2 A + n^2 \sin^2 A - 2mn \sin A \cos A = q^2$ .... (4)

Adding (3) and (4) we get,

$m^2(\sin^2 A + \cos^2 A) + n^2(\sin^2 A + \cos^2 A) = p^2 + q^2$

$\Rightarrow m^2 + n^2 = p^2 + q^2$ [$\because \sin^2 A + \cos^2 A = 1$]

OR

According to the question,

$\sec\theta=\cos ec 60^\circ$

$\Rightarrow \sec\theta=\cos ec\left(90^\circ-30^\circ\right)$

$\Rightarrow \sec\theta=\sec 30^\circ$ $[\cos ec\left(90^\circ-\theta\right)=\sec\theta]$

$\Rightarrow \theta=30^\circ$

$\therefore 2\cos^2\theta-1$

$=2\cos^2 30^\circ-1$

$=2\left(\frac{\sqrt{3}}{2}\right)^2-1$

$=2\times\frac{3}{4}-1$

$= \frac{3}{2} - 1$
$= \frac{3-2}{2}$
$= \frac{1}{2}$

25. Guven: A DABC in which D is the mid-point of AB and DE $\parallel$ BC.

To prove: E is the mid-point of AC

Proof:

$\therefore DE||BC$

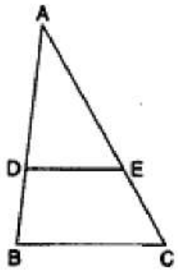
$\therefore \frac{AD}{DB} = \frac{AE}{EC}$ (1)......... [By basic proportionality theorem]

$\because$ D is the midpoint of AB

$\therefore AD = DB \therefore \frac{AD}{DB} = 1$

$\therefore \frac{AE}{EC} = 1......From(1)$

$\therefore AE = EC \therefore$ E is the mid-point of AC.



OR

Since $\triangle$ NSQ $\cong$ $\triangle$MTR

$\therefore \angle$ SQN = $\angle$ TRM

$\Rightarrow \angle$ Q= $\angle$ R(in $\triangle$PQR)

$\angle Q$ = 90^0 - $\frac{1}{2}\angle P$

Again 1 = $\angle$ 2 [given in $\triangle$ PST](Isosceles property)

$\therefore \angle$ 1= $\angle$ 2 = $\frac{1}{2}(180^\circ - \angle P)$

$= 90^\circ - \frac{1}{2}\angle P$

Thus, in $\triangle$PTS and $\triangle$PRQ

$\angle$ 1 = $\angle$ Q [Each = 90° - $\frac{1}{2}\angle P$]

$\angle$ 2 = $\angle$ R, $\angle$ P= $\angle$ P(Common)

$\triangle$ PTS $\cong$ $\triangle$PRQ

Section C

26. Let the total number of swans be x.

$\therefore$Number of swans playing on the shore of the tank = $\frac{7}{2}\sqrt{x}$

It is given that there are two remaining swans playing in the water. Hence, total no. of swans = $\frac{7}{2}\sqrt{x}$ +2, which is equal to x.

$Clearly; \quad x = \frac{7}{2}\sqrt{x} + 2$

$\Rightarrow \quad x - \frac{7}{2}\sqrt{x} - 2 = 0$

$\Rightarrow \quad y^2 - \frac{7}{2}y - 2 = 0$, where $y = \sqrt{x} \Rightarrow y^2 = x$

$\Rightarrow$ 2y^2 - 7y - 4 = 0

$\Rightarrow$ 2y^2 - 8y + y - 4 = 0

$\Rightarrow$ 2y(y - 4) + (y - 4) = 0

$\Rightarrow$ (y - 4) (2y +1) = 0

$\Rightarrow$ y = 4 or, y = $-\frac{1}{2}$

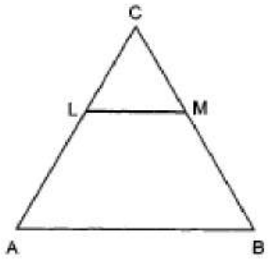
$\Rightarrow$ y = 4 [$\because$ y = $-\frac{1}{2}$ is not possible as y is square root of x]

$\Rightarrow$ x = y^2 $\Rightarrow x = 4^2 = 16$

Hence, the total number of swans = x = 16.

27. We have, AL = x - 3, AC = 2x, BM = x - 2 and BC = 2x + 3, and we need to find the value of x.

In ΔABC, we have



$LM \| AB$

$\therefore \quad \frac{AL}{LC} = \frac{BM}{MC}$ [By Thaley's Theorem]

$\Rightarrow \quad \frac{AL}{AC-AL} = \frac{BM}{BC-BM}$

$\Rightarrow \quad \frac{x-3}{2x-(x-3)} = \frac{x-2}{(2x+3)-(x-2)}$

$\Rightarrow \quad \frac{x-3}{x+3} = \frac{x-2}{x+5}$

$\Rightarrow$ (x - 3) (x + 5) = (x - 2) (x + 3)

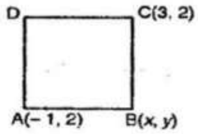
$\Rightarrow x^2 + 2x - 15 = x^2 + x - 6$

$\Rightarrow$ x = 9

28. Let ABCD be a square and B (x, y) be the unknown vertex.

AB = BC

$\Rightarrow AB^2 = BC^2$



$\Rightarrow (x + 1)^2 + (y - 2)^2 = (x - 3)^2 + (y - 2)^2$

$\Rightarrow x^2 + 1 + 2x + y^2 + 4 - 4x = x^2 - 6x + 9 + y^2 + 4 - 4x$

$\Rightarrow$ 2x + 1 = - 6x + 9

$\Rightarrow$ 8x = 8

$\Rightarrow$ x = 1 .... (i)

In $\triangle$ABC, $AB^2 + BC^2 = AC^2$

$\Rightarrow (x + 1)^2 + (y - 2)^2 + (x - 3)^2 + (y - 2)^2 = (3 + 1)^2 + (2 - 2)^2$

$\Rightarrow x^2 + 1 + 2x + y^2 - 4x + 4 + x^2 + 9 - 6x + y^2 + 4 - 2y = 16 + 0$

$\Rightarrow 2x^2 + 2y^2 + 2x - 4y - 6x - 4y + 1 + 4 + 9 + 4 = 16$

$\Rightarrow 2x^2 + 2y^2 - 4x - 8y + 2 = 0$

$\Rightarrow x^2 + y^2 - 2x - 4y + 1 = 0$ ..... (ii)

Putting the value of x in eq. (ii),

$1 + y^2 - 2 - 4y + 1 = 0$

$\Rightarrow y^2 - 4y = 0$

$\Rightarrow$ y(y - 4) = 0

$\Rightarrow$ y = 0 or 4

Hence the other vertices are (1, 0) and (1, 4).

OR

Let the coordinates of A be (x, y) which lies on line joining P(6, -6) and Q (-4, -1)

such that $\frac{\text{PA}}{\text{PQ}} = \frac{2}{5}$

$\Rightarrow \frac{\text{PA}}{\text{PQ}-\text{PA}} = \frac{2}{5-2}$

$\Rightarrow \frac{\text{PA}}{\text{AQ}} = \frac{2}{3}$

$\Rightarrow \text{PA} : \text{AQ} = 2 : 3$

Now by section formula x and y becomes as shown below

Since, P(6, -6) and Q(-4, -1)

$\therefore x = \frac{mx_2+nx_1}{m+n} = \frac{2(-4)+3\times6}{2+3}$

$= \frac{-8+18}{5} = \frac{10}{5} = 2$

$y = \frac{my_2+ny_1}{m+n} = \frac{2\times(-1)+3(-6)}{2+3}$

$= \frac{-2-18}{5} = \frac{-20}{5} = -4$

Therefore Coordinates of A are (2, -4). As A lies on line segment joining the points P and Q so it must satisfy equation of line segment.

Therefore Substituting the value of x and y i.e; value of A (2,-4) in 3x + k(y + 1) =0

⇒ 3 × 2 + k(-4 + 1) = 0 ⇒ 6 - 3k =0

$\Rightarrow 3k = 6 \Rightarrow k = \frac{6}{3} = 2$

29. Let first we consider, $\sqrt{2}\frac{1}{3}$ be rational. We can write $\sqrt{2}/3$ as

$\frac{1}{3} \times \sqrt{2}$

We know that product of two rational number is always a rational number.

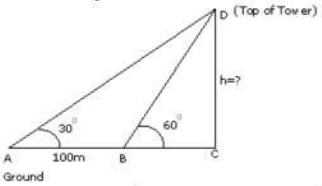
$\frac{\sqrt{2}}{3} \times 3 = \sqrt{2}$

But

$\sqrt{2}$ is irrational.

∴ Here the Contradiction arises by assuming that $\frac{\sqrt{2}}{3}$ is rational. Actually it is irrational.

Hence, $\frac{\sqrt{2}}{3}$ is irrational.

30. 

In $\triangle BCD, \frac{h}{x} = \tan 60^\circ = \sqrt{3}$ (BC=x)

$h = \sqrt{3x}$ ....(i)

In $\triangle ACD, \frac{h}{100+x} = \tan 30^\circ = \frac{1}{\sqrt{3}}$

$\Rightarrow h\sqrt{3} = 100 + x$

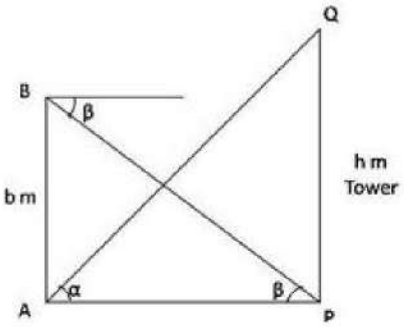
$\Rightarrow h\sqrt{3} = 100 + \frac{h}{\sqrt{3}}$

$\Rightarrow h\left[\sqrt{3} - \frac{1}{\sqrt{3}}\right] = 100$

$\Rightarrow h\left[\frac{3-1}{\sqrt{3}}\right] = 100$

$\Rightarrow h = \frac{100\sqrt{3}}{2} = 50\sqrt{3} = 50 \times 1.732 = 86.6\text{m}$

OR



Let height of tower = QP = h m

In ΔBAP

$\tan\beta = \frac{BA}{AP}$

$\Rightarrow \tan\beta = \frac{b}{AP}$

$\Rightarrow AP = \frac{b}{\tan\beta}$

$\Rightarrow AP = b \times \cot\beta$ .... (i)

In ΔQPA

$\tan\alpha = \frac{QP}{AP}$

$\Rightarrow QP = AP \times \tan\alpha$

$\Rightarrow QP = b\cot\beta \times \tan\alpha$ From (i)

31. First, we will convert the graph given into tabular form as shown below:

Class interval	Frequency (f_i)	Mid value (x_i)	f_ix_i	Cumulative Frequency
1 – 4	6	2.5	15	6
4 – 7	30	5.5	165	36
7 – 10	40	8.5	340	76
10 – 13	16	11.5	184	92
13 – 16	4	14.5	58	96
16 – 19	4	17.5	70	100
	N = $\sum f_i$ = 100		Σf_ix_i = 832	

i. N = 100

Mean = $\frac{\Sigma f_ix_i}{N} = \frac{832}{100}$ = 8.32

ii. $\frac{N}{2} = \frac{100}{2}$ = 50

The cumulative frequency just greater than $\frac{N}{2}$ is 76, then the median class is 7 - 10 such that

l = 7, h = 10 - 7 = 3, f = 40, F = 36

Median = l + $\frac{\frac{N}{2}-F}{f} \times$ h

= 7 + $\frac{50-36}{40} \times$ 3

$= 7 + \frac{42}{40}$ = 7 + 1.05 = 8.05

iii. Mode = 3 Median - 2 Mean

= 3 $\times$ 8.05 - 2 $\times$ 8.32 = 7.51

Section D

32. Let x and y be the numerator and the denominator of the fraction.

According to the question,

x + y = 8...(1)

Also, we have,

$\frac{x+3}{y+3} = \frac{3}{4}$

⇒ 4(x + 3) = 3(y + 3)

⇒ 4 x + 12 = 3y + 9

⇒ 4x - 3y = - 3...(2)

Multiplying eq (1) by 4, we get,

4x + 4y = 32 ...(3)

Subtracting eq (2) from eq (3), we get,

7y = 35

Thus, y = 5

Substitute the value of y in eq (1), we get, x = 3.

Thus, we have x = 3 and y = 5.

Hence, the required fraction is $\frac{3}{5}$.

OR

Suppose, speed of the train be x km/hr and the speed of taxi be y km/h.

time taken to cover 300 km by the train = $\frac{300}{x}$ hours

time taken to cover 200 km by the taxi = $\frac{200}{y}$ hours

Total time taken = $5\frac{30}{60}$ hours $= 5\frac{1}{2}$ hours $= \frac{11}{2}$ hours

$\therefore \frac{300}{x} + \frac{200}{y} = \frac{11}{2}$

$\Rightarrow \frac{600}{x} + \frac{400}{y} = 11$

Put $\frac{1}{x}$ = u and $\frac{1}{y}$ = v

$\Rightarrow 600u + 400v = 11$ ....(i)

time taken to cover $260\ km$ by the train = $\frac{260}{x}$ hours

time taken to cover $240\ km$ by the taxi = $\frac{240}{y}$ hours

Total time taken = $5\frac{36}{60}$ hours $= 5\frac{1}{2}$ hours $= \frac{11}{2}$ hours

$\Rightarrow 1300u + 1200v = 28$ ....(ii)

Multiplying (i) by 3 and subtracting (ii) from it,

$\Rightarrow 500u = 5 \Rightarrow u = \frac{5}{500} \Rightarrow u = \frac{1}{100}$

Substituting u = $\frac{1}{100}$ in (i), $\Rightarrow$ v = $\frac{1}{80}$

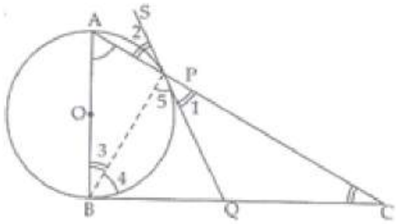
$\therefore u = \frac{1}{100} \Rightarrow \frac{1}{x} = \frac{1}{100} \Rightarrow x = 100$

$v = \frac{1}{80} \Rightarrow \frac{1}{y} = \frac{1}{80} \Rightarrow y = 80$

$\therefore$ the speed of the train = 100 km/hr

the speed of the taxi = 80 km/hr

33. **Given:** ΔABC in which ∠B = 90°



Circle with diameter AB intersect the hypotenuse AC at P.

A tangent SPQ at P is drawn to meet BC at Q.

To prove: Q is mid point of BC.

Construction: Join PB.

Proof: SPQ is tangent and AP is chord at contact point P.

Therefore,∠2 = ∠3 [since Angles in alternate segment of circle are equal]

∠2 = ∠1 [Vertically opposite angles]

∠3 = ∠1 …(i) [From above two relations]

∠ABC = 90° [Given]

OB is radius , therefore BC will be tangent at B.

Therefore,∠3 = 90° - ∠4 …(ii)

∠APB = 90° [∠ in a semi circle]

⇒ ∠C = 90° - ∠4....(iii)

From (ii) and (iii), ∠C = ∠3

Using (i), ∠C = ∠1

⇒ CQ = QP …(iv) [Sides opp. to = ∠s in ΔQPC]

∠4 = 90° - ∠3 [From fig.]

∠5 = 90° - ∠1

∠3 = ∠1

Therefore,∠4 = ∠5

⇒ PQ = BQ …(v) [Sides opp. to equal angles in ΔQPB]

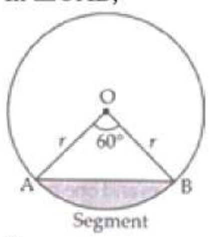
From (iv) and (v),

BQ = CQ

Therefore, Q is mid-point of BC. Hence, proved.

34. Area of minor segment = Area of sector – Area of △OAB

In △OAB,



θ = 60°

OA = OB = r = 12 cm

∠B = ∠A = x [∠s opp. to equal sides are equal]

⇒ ∠A + ∠B + ∠O = 180°

⇒ x + x + 60° = 180°

⇒ 2x = 180° – 60°

$\Rightarrow x = \frac{120°}{2} = 60°$

$\therefore \triangle OAB$ is equilateral $\triangle$ with each side (a) = 12 cm

Area of the equilateral $\triangle = \frac{\sqrt{3}}{4}a^2$

Area of minor segment = Area of the sector – Area of $\triangle OAB$

$= \frac{\pi r^2\theta}{360°} - \frac{\sqrt{3}}{4}a^2$

$= \frac{3.14\times12\times12\times60°}{360°} - \frac{\sqrt{3}}{4}\times12\times12$

$= 6.28 \times 12 - 36\sqrt{3}$

$\therefore$ Area of minor segment = $(75.36 - 36\sqrt{3})$ cm^2.

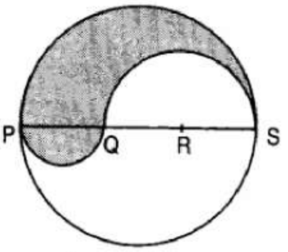
OR

PS = Diameter of a circle of radius 6 cm = 12 cm

$\therefore$ PQ = QR = RS = $\frac{12}{3}$ = 4cm, QS = QR + RS = (4 + 4)cm = 8 cm

Let P be the perimeter and A be the area of the shaded region.

P = Arc of semi-circle of radius 6 cm + Arc of semi-circle of radius 4 cm + Arc of semi-circle of radius 2 cm



$\Rightarrow P = (\pi\times6 + \pi\times4 + \pi\times2)\text{cm} = 12\pi\text{cm}$

and, A = Area of semi-circle with PS as diameter + Area of semi-circle with PQ as diameter - Area of semi-circle with QS as diameter.

$\Rightarrow A = \frac{1}{2}\times\frac{22}{7}\times(6)^2 + \frac{1}{2}\times\frac{22}{7}\times2^2 - \frac{1}{2}\times\frac{22}{7}\times(4)^2$

$\Rightarrow A = \frac{1}{2}\times\frac{22}{7}(6^2 + 2^2 - 4^2) = \frac{1}{2}\times\frac{22}{7}\times24 = \frac{264}{7}\text{cm}^2$ = 37.71 cm^2

35. Total no. of outcomes = (1,1), (1,4), (1,9), (1,16), (2,1), (2,4), (2,9), (2,16), (3,1), (3,4), (3,9), (3,16), (4,1), (4,4), (4,9), (4,16)

Total possible outcome = 16

Total favorable event having product less than 16 = (1,1), (1,4), (1,9), (2,1), (2,4), (3,1), (3,4), (4,1)

Probability = $\frac{\text{Favourable event outcome}}{\text{Total event}}$

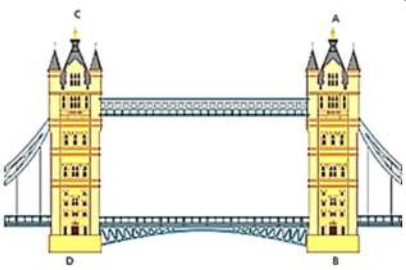
P(E) = $\frac{8}{16} = \frac{1}{2}$

Section E

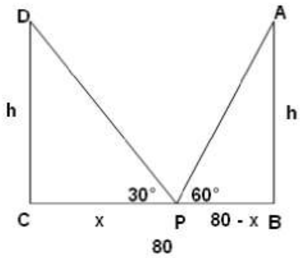
36. **Read the text carefully and answer the questions:**

Tower Bridge is a Grade I listed combined bascule and suspension bridge in London, built between 1886 and 1894, designed by Horace Jones and engineered by John Wolfe Barry. The bridge is 800 feet (240 m) in length and consists of two bridge towers connected at the upper level by two horizontal walkways, and a central pair of bascules that can open to allow shipping.

In this bridge, two towers of equal heights are standing opposite each other on either side of the road, which is 80 m wide. During summer holidays, Neeta visited the tower bridge. She stood at some point on the road between these towers. From that point between the towers on the road, the angles of elevation of the top of the towers was 60^o and 30^o respectively.



(i)



Suppose AB and CD are the two towers of equal height h m. BC be the 80 m wide road. P is any point on the road. Let CP be x m, therefore BP = (80 – x).

Also, $\angle$APB = 60^o and $\angle$DPC = 30^o

In right angled triangle DCP,

tan 30^o = $\frac{CD}{CP}$

$\Rightarrow \frac{h}{x} = \frac{1}{\sqrt{3}}$

$\Rightarrow h = \frac{x}{\sqrt{3}}$ ....(i)

In right angled triangle ABP,

tan 60^o = $\frac{AB}{AP}$

$\Rightarrow \frac{h}{80-x} = \sqrt{3}$

$\Rightarrow h = \sqrt{3}(80 - x)$

$\Rightarrow \frac{x}{\sqrt{3}} = \sqrt{3}(80 - x)$

$\Rightarrow$ x = 3(80 – x)

$\Rightarrow$ x = 240 – 3x

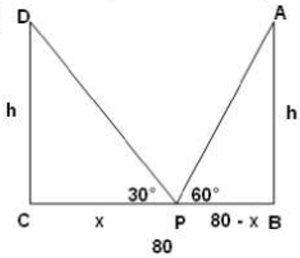
$\Rightarrow$ x + 3x = 240

$\Rightarrow$ 4x = 240

$\Rightarrow$ x = 60

Thus, the position of the point P is 60 m from C.

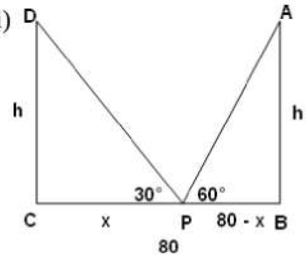
(ii)



Height of the tower, h = $\frac{x}{\sqrt{3}} = \frac{60}{\sqrt{3}}$ = $20\sqrt{3}$

The height of each tower is $20\sqrt{3}$ m.

(iii)



The distance between Neeta and top of tower AB.

In $\triangle$ABP

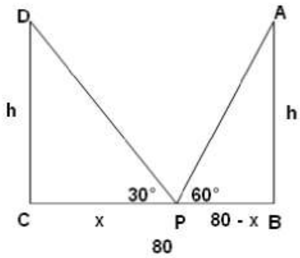
sin 60^o = $\frac{AB}{AP}$

$\Rightarrow AP = \frac{AB}{\sin 60^0}$

$\Rightarrow AP = \frac{20\sqrt{3}}{\frac{\sqrt{3}}{2}}$

$\Rightarrow$ AP = 40 m

OR



The distance between Neeta and top of tower CD.

In $\triangle$CDP

$\sin 30^0 = \frac{CD}{PD}$

$\Rightarrow PD = \frac{CD}{\sin 30^0}$

$\Rightarrow PD = \frac{20\sqrt{3}}{\frac{1}{2}} = 40\sqrt{3}$

$\Rightarrow PD = 40\sqrt{3}$

37. **Read the text carefully and answer the questions:**

Sehaj Batra gets pocket money from his father every day. Out of pocket money, he saves money for poor people in his locality. On 1st day he saves ₹27.5 On each succeeding day he increases his saving by ₹2.5.

(i) Money saved on 1st day = ₹27.5

$\because$ Sehaj increases his saving by a fixed amount of ₹2.5

$\therefore$ His saving form an AP with a = 27.5 and d = 2.5

$\therefore$ Money saved on 10th day,

a_{10} = a + 9d = 27.5 + 9(2.5)

= 27.5 + 22.5 = ₹50

(ii) a_{25} = a = 24d

= 27.5 + 24(2.5)

= 27.5 + 60 = ₹ 87.5

OR

Let S_n = 387.5, a = 27.5 and d = 2.5

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow 387.5 = \frac{n}{2}[2 \times 27.5 + (n-1)2.5]$

$\Rightarrow 387.5 = \frac{n}{2}[55 + (n-1) \times 2.5]$

$\Rightarrow$ 775 = 55n + 2.5n^2 - 2.5n

$\Rightarrow$ 25n^2 + 525n = 7750 = 0

$\Rightarrow$ n^2 + 21n - 310 = 0

$\Rightarrow$ (n + 31)(n - 10) = 0

$\Rightarrow$ n = -31 reject n = 10 accept

So in 10 years Sehaj saves ₹ 387.5.

(iii) Total amount saved by Sehaj in 30 days.

$= \frac{30}{2}[2 \times 27.5 + (30-1) \times 2.5]$

= 15(55 + 29(2.5)

= ₹1912.5

38. **Read the text carefully and answer the questions:**

Ashish is a Class IX student. His class teacher Mrs Verma arranged a historical trip to great Stupa of Sanchi. She explained that Stupa of Sanchi is great example of architecture in India. Its base part is cylindrical in shape. The dome of this stupa is hemispherical in shape, known as Anda. It also contains a cubical shape part called Hermika at the top. Path around Anda is known as Pradakshina Path.

(i) Volume of Hermika = side^3 = 10 $\times$ 10 $\times$ 10 = 1000 m^3

(ii) r = radius of cylinder = 24, h = height = 16

Volume of cylinder = $\pi r^2 h$

$\Rightarrow$ V = $\frac{22}{7} \times 24 \times 24 \times 14$ = 25344 m^3

OR

Since Anda is hespherical in shape r = radius = 21

V = Volume of Anda = $\frac{2}{3} \times \pi \times r^3$

$\Rightarrow$ V = $\frac{2}{3} \times \frac{22}{7} \times 21 \times 21 \times 21$

$\Rightarrow$ V = 44 $\times$ 21 $\times$ 21 = 19404 m^3

(iii) Volume of brick = 0.01 m^3

$\Rightarrow$ n = Number of bricks used for making cylindrical base = $\frac{\text{Volume of cylinder}}{\text{Volume of one brick}}$

$\Rightarrow n = \frac{25344}{0.01}$ = 2534400